Michigan State University
STT 455 - Actuarial Models I
Final Examination
Tuesday, 10 December 2013  5:45 - 7:45 PM
Total Score: 100 points

Name: Suggested Solutions  Section 2

- There are ten (10) multiple choice questions here and you are to answer all questions asked. Each question is worth 10 points.

- Please double check your work as no partial points will be granted.

- Please write legibly.

- The Illustrative Life Table (ILT) is attached in the last two pages of this paper.

- Anyone caught writing after time has expired will be given a mark of zero.

- Good luck.

- Have a Happy and Healthy Christmas and New Year!

<table>
<thead>
<tr>
<th>Question</th>
<th>Worth</th>
<th>Score</th>
</tr>
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<tbody>
<tr>
<td>1</td>
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<td>C</td>
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<td>10</td>
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</table>
Question No. 1: (10 points)

For a fully continuous whole life insurance of $1 issued to (40), you are given:

- Mortality follows De Moivre's (or Uniform distribution) law with $\omega = 100$.
- $\delta = 0.05$
- Premium, based on the Equivalence Principle, is paid continuously at the annual rate of P.

Calculate $P$.

\[ APV(FP_0) = APV(FB_0) \]

\[ P \bar{A}_{40} = \bar{A}_{40} \Rightarrow P = \frac{\bar{A}_{40}}{\bar{a}_{40}} \]

(a) 0.015

(b) 0.021

(c) 0.023

When you have De Moivre's, it is easier to work with insurances. So, we have

(d) 0.025

\[ \bar{A}_{40} = \int_0^{60} e^{-0.05t} \frac{1}{60} dt = \frac{1}{60} \cdot \frac{1}{0.05} (1 - e^{-0.05(60)}) \]

\[ = \frac{1}{3} (1 - e^{-3}) = 0.3167376 \]

\[ P = \frac{\bar{A}_{40}}{\bar{a}_{40}} = \frac{\bar{A}_{40}}{(-\bar{A}_{40})/\delta} = \delta \frac{\bar{A}_{40}}{1-\bar{A}_{40}} \]

\[ = 0.05 \frac{0.3167376}{1 - 0.3167376} \]

\[ = 0.2317833 \]
Question No. 2: (10 points)

A fully discrete whole life insurance of $100 is issued to (46). You are given:

- Expenses consist of 10% of annual gross premium in the first year and 4% in subsequent years.
- $A_{46} = 0.15$
- $p_{45} = 0.99$
- $i = 0.04$

Let $G$ be the annual gross premium.

Calculate the annual gross premium for this policy.

(a) $0.67$

(b) $0.70$

(c) $0.73$

(d) $0.77$

(e) $0.80$

\[
APV(FP_0) = APV(FB_0) + APV(FE_0)
\]

\[
G \cdot A_{46} = 100A_{46} + 0.06G + 0.04G \cdot A_{46}
\]

\[
G(0.96 \cdot A_{46} - 0.06) = 100A_{46}
\]

\[
G = \frac{100A_{46}}{0.96 \cdot A_{46} - 0.06}
\]

Now apply recursion to derive $A_{46}$

\[
A_{45} = \frac{v}{q_{45} + p_{45}} A_{46} \implies A_{46} = \frac{A_{45} - vq_{45}}{vp_{45}}
\]

\[
= \frac{0.15 - (0.04)(1 - 0.99)}{(0.04)(0.99)} = 0.1474747
\]

And use relationship

\[
\ddot{A}_{46} = \frac{1 - A_{46}}{d} = \frac{1 - 0.1474747}{0.04/0.04} = 22.16566
\]

Finally, plug values

\[
G = \frac{100(0.1474747)}{0.96(22.16566) - 0.06} = 0.6950117 \approx 0.70
\]
Question No. 3: (10 points)

For a special fully discrete whole life insurance issued to (50), you are given:

- The death benefit is $1,000 plus the return of all premiums paid without interest.
- \( i = 0.05 \)
- \((IA)_{50} = 9.268\)
- Based on the Equivalence Principle, the level annual premium for this insurance is equal to $38.491.

Calculate \( \ddot{a}_{50} \).

\[
\text{APV}(FP_0) = \text{APV}(FB_0)
\]

\[
P \ddot{a}_{50} = 1000 \frac{A_{50}}{1 - d \ddot{a}_{50}} + P(IA)_{50}
\]

Rearranging, we get

\[
\ddot{a}_{50} \left( P + 1000d \right) = 1000 + P(IA)_{50}
\]

\[
\ddot{a}_{50} = \frac{1000 + P(IA)_{50}}{P + 1000d}
\]

\[
= \frac{1000 + (38.491)(9.268)}{38.491 + 1000 (0.05/1.05)}
\]

\[
= 15.75582 \approx 15.8
\]
Question No. 4: (10 points)

For a special type of whole life insurance issued to (30), you are given:

- Death benefits are 5,000 for the first 10 years and 1,000 thereafter.
- Death benefits are payable at the moment of death.
- Deaths are uniformly distributed over each year of age interval.
- \( i = 5\% \)

- The following table of actuarial present values:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 1000A_x )</th>
<th>( 1000E_x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>112.31</td>
<td>779.79</td>
</tr>
<tr>
<td>35</td>
<td>138.72</td>
<td>779.20</td>
</tr>
<tr>
<td>40</td>
<td>171.93</td>
<td>777.14</td>
</tr>
</tbody>
</table>

Calculate the Actuarial Present Value (APV) of the benefits for this policy.

(a) 25.90
(b) 147.25
(c) 399.28
(d) 438.08
(e) 468.42

\[
\text{APV(benefits)} = 5000 \frac{1}{1.05} A_{30} - 4000 \frac{1}{1.05} E_{30} A_{40} \\
= 5000 \frac{1}{1.05} (1.1231) - 4000 (1.77979)(1.77720) \\
* (1.17173) \\
= 147.2457 \approx 147.25
\]
Question No. 5: (10 points)

You are given:

- \( p_x = 0.99 \)
- \( p_{x+1} = 0.98 \)
- \( p_{x+2} = 0.96 \)
- \( q_x = 0.89 \)
- \( q_{x+1} = 0.92 \)

Calculate \( q_{x+1} \).

(a) 0.960
(b) 0.963
(c) 0.966
(d) 0.969
(e) 0.972

\[
3p_{x+1} = 2p_{x+1} \cdot p_{x+3} \quad \text{(per 3, 2 on left)}
\]

\[
p_x \cdot p_{x+1} \cdot p_{x+2} \cdot p_{x+3} = 4p_x
\]

\[
p_{x+3} = \frac{4p_x}{p_x \cdot p_{x+1} \cdot p_{x+2}} = \frac{0.89}{0.99 \cdot 0.98 \cdot 0.96}
\]

\[
= 0.955559
\]

\[
2p_{x+1} = \frac{3p_{x+1}}{p_{x+3}}
\]

\[
= \frac{0.92}{0.955559}
\]

\[
= 0.9627872 \approx 0.963
\]
Question No. 6: (10 points)

You are given:

- $A_{x+20} = 0.40$
- $\overline{20}E_x = 0.50$
- $A_{x:20} = 0.55$
- $i = 0.03$

Calculate $A_x$.

(a) 0.05

(b) 0.15

(c) 0.25

(d) 0.40

(e) 0.50

\[
A_x = A_{x:20} + \overline{20}E_x A_{x+20} \\
= A_{x:20} - \overline{20}E_x + \overline{20}E_x A_{x+20} \\
= A_{x:20} - \overline{20}E_x (1 - A_{x+20}) \\
= 0.55 - 0.50 (1 - 0.40) \\
= 0.25
\]
Question No. 7: (10 points)

Get-a-Life Insurance Company sells 10,000 fully discrete whole life insurance policies of $1, each with the same age 50. You are given:

- All policies have independent future lifetime.
- $A_{50} = 0.300$
- $\overline{A}_{50} = 0.125$
- $i = 0.05$
- Premium is determined according to the portfolio percentile principle, with the probability that the total future loss on the portfolio is negative at least 95%.
- The 95th percentile of a standard Normal distribution is 1.645.

Calculate the annual premium for each policy.

(a) 0.0204
(b) 0.0207
(c) 0.0210
(d) 0.0213
(e) 0.0216

\[
\begin{align*}
L_{aqg} &= \sum_{i=1}^{10,000} L_{0,i} \quad \text{where } L_{0,i} = V^{k+1} - P \overline{A}^{k+1} \\
&= V \left(1 + \frac{P}{d}\right) - \frac{P}{d} - \frac{1.300}{0.85*0.05} \\
&= 0.3 - 14.7P \\
\end{align*}
\]

\[
\begin{align*}
\text{Var}[L_{0,i}] &= (1 + \frac{P}{d})^2 (\overline{A}_{50}^2 - \overline{A}_{50}^2) \\
&= (1 + \frac{P}{d})^2 (0.25 - 0.3^2) \\
&= (1 + \frac{P}{d})^2 (0.035) \\
\end{align*}
\]

\[
\begin{align*}
E[L_{aqg}] &= 10,000 (0.3 - 14.7P) \\
\text{Var}[L_{aqg}] &= 10,000 (1 + \frac{P}{d})^2 (0.035) \\
\end{align*}
\]

\[
\begin{align*}
P_i[L_{aqg} < 0] \geq 0.95 \implies P\left[Z < \frac{-10,000(0.3 - 14.7P)}{\sqrt{10,000(1 + \frac{P}{d})^2 (0.035)}} \right] \geq 0.95 \\
\end{align*}
\]

Solving for $P$, with $d = 0.05$

\[
\begin{align*}
1.645 &\geq 1.645 \sqrt{1.05} - 10,000(0.3 - 14.7P) \\
\end{align*}
\]

\[
\begin{align*}
&\geq \left(14,700 - \frac{1.645(100)}{\sqrt{1.05}}\right) + 3000 \\
&\geq 14,700 - \frac{1.645(100)}{\sqrt{1.05}} + 3000 \\
&\geq 14,700 - 14,700 \times \frac{1.645}{\sqrt{1.05}} + 3000 \\
&\geq 0.020788
\end{align*}
\]
Question No. 8: (10 points)

Consider a life \((x)\) with curtate future lifetime denoted by \(K\). A fully discrete whole life insurance is issued to \((x)\) where:

- The death benefit is $100.
- Expenses, to be paid at the beginning of each year, consist of 4% of each premium.
- The annual premium is \(G\).
- Denote the discount rate by \(d = \frac{i}{1+i}\).

Which of the following is the loss-at-issue random variable?

\[
\begin{align*}
(a) \quad & \left(100 + \frac{1.04G}{d}\right) v^{K+1} - \frac{1.04G}{d} \\
(b) \quad & \left(100 - \frac{1.04G}{d}\right) v^{K+1} + \frac{1.04G}{d} \\
(c) \quad & 100 v^{K+1} - 1.04 G \ddot{a}_{K+1} \\
(d) \quad & \left(100 - \frac{0.96G}{d}\right) v^{K+1} + \frac{0.96G}{d} \\
(e) \quad & \left(100 + \frac{0.96G}{d}\right) v^{K+1} - \frac{0.96G}{d}
\end{align*}
\]

\[
L_0 = PVFB_o + PVFE_o - PVFP_o \\
= 100 v^{K+1} + 0.04 G \ddot{a}_{k+1} - G \ddot{a}_{k+1} \\
= 100 v^{K+1} - 0.96 G \ddot{a}_{k+1} \\
\frac{1 - v^{K+1}}{d}
\]

\[
= \left(100 + \frac{0.96G}{d}\right) v^{K+1} - \frac{0.96G}{d}
\]
Question No. 9: (10 points)

You are given:

- $\bar{a}_x = 3.65$
- $\bar{a}_{x+1} = 3.55$
- $p_x = 0.80$

Calculate $i$.

(a) 2%

(b) 4%

(c) 7%

(d) 15%

(e) 20%

Use reversion formula:

$$\bar{a}_x = 1 + \sqrt{p_x \cdot \bar{a}_{x+1}}$$

$$\frac{\bar{a}_x - 1}{\bar{p}_x \cdot \bar{a}_{x+1}} = \sqrt{p_x} \Rightarrow i = \frac{p_x \cdot \bar{a}_{x+1}}{\bar{a}_x - 1} - 1$$

$$= \frac{0.8 \cdot 3.55}{3.65 - 1} - 1$$

$$= 0.0716981$$

(c) 7%
Question No. 10: (10 points)

A fully discrete whole life policy of $100 is issued to (50). Level annual premium is determined with the following expense assumptions:

<table>
<thead>
<tr>
<th></th>
<th>% of Premium</th>
<th>Per 100</th>
<th>Per Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>First year</td>
<td>20%</td>
<td>0.12</td>
<td>2.0</td>
</tr>
<tr>
<td>Renewal years</td>
<td>5%</td>
<td>0.07</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Mortality follows the Illustrative Life Table with interest rate \( i = 6\% \).

Calculate the gross annual premium for this policy.

(a) 2.0

\[ \text{APV}(FP_0) = \text{APV}(FB_0) + \text{APV}(FE_0) \]

(b) 2.6

\[ GG_{50} = 100 \ddot{A}_{50} + 0.15G + 0.05G \ddot{A}_{50} + 0.05 + 0.07 \ddot{A}_{50} + 1 + \ddot{A}_{50} \]

(c) 3.2

(d) 3.8

(e) 4.4

Rearranging, we get

\[ G(0.95 \ddot{A}_{50} - 0.15) = 100 \ddot{A}_{50} + 1.05 + 1.07 \ddot{A}_{50} \]

\[ G = \frac{100 \ddot{A}_{50} + 1.05 + 1.07 \ddot{A}_{50}}{0.95 \ddot{A}_{50} - 0.15} \]

\[ = 100 (0.24905) + 1.05 + 1.07 (13.2668) \]

\[ = 3.224042 \]