Please follow the instructions below:

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Submit your work to our graduate assistant, Ed Cruz, at C505 Wells.

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I certify that this is my own work, and that I have not copied the work of another student.

Signature: ___________________________  Date: ___________________________
1. (30 points) Let $T_0$ be the lifetime of a newborn random variable with density function defined by

$$f_0(x) = \frac{1}{100} e^{-x/50} + \frac{1}{200} e^{-x/100}, \quad \text{for } x \geq 0.$$ 

Find expressions for the following:

(a) [7 points] $S_0(x)$

(b) [7 points] $\mu_x$

(c) [7 points] $\varphi_x$

(d) [9 points] Calculate $25\% q_{40}$ and interpret this probability.

(a) $S_0(x) = \int_x^\infty f_0(z) \, dz = \frac{1}{2} \int_x^\infty \frac{1}{50} e^{-z/50} \, dz + \frac{1}{2} \int_x^\infty \frac{1}{100} e^{-z/100} \, dz$

$$= \frac{1}{2} e^{-x/50} + \frac{1}{2} e^{-x/100}$$

(b) $\mu_x = \frac{f_0(x)}{S_0(x)} = \frac{\frac{1}{50} e^{-x/50} + \frac{1}{100} e^{-x/100}}{e^{-x/50} + e^{-x/100}}$

(c) $\varphi_x = \frac{S_0(x+t)}{S_0(x)} = \frac{e^{-(x+t)/50} + e^{-(x+t)/100}}{e^{-x/50} + e^{-x/100}}$

(d) $25\% q_{40} = 25\% q_{40} - 30\% q_{40}$

$$= \frac{65/50 - 65/100}{e^{-40/50} + e^{-40/100}} - \frac{70/50 - 70/100}{e^{-40/50} + e^{-40/100}}$$

$$= 0.0459$$

This gives the probability that a 40-year-old will die between the ages of 65 and 70.
2. (30 points) You are given:

\[ 10p_x = 0.90 \]
\[ 15p_x = 0.85 \]
\[ 5p_{x+5} = 0.95 \]

Calculate the following:

(a) [10 points] \( 5q_x \)
(b) [10 points] \( 10/5 q_x \)
(c) [10 points] \( 10p_{x+5} \)

\[(a)\]
\[
10p_x = 5f_x \cdot 5f_{x+5} \Rightarrow 5f_x = \frac{10p_x}{5f_{x+5}} = \frac{0.90}{0.95}
\]
\[
\Rightarrow 5q_x = 1 - 5p_x = 1 - \frac{0.90}{0.95} = \frac{0.5}{0.95} = \frac{5}{9.5} = 0.526
\]

\[(b)\]
\[
15p_x = 10p_x \cdot 5f_{x+10} \Rightarrow 5f_{x+10} = \frac{15p_x}{10p_x} = \frac{0.85}{0.90} = \frac{17}{18}
\]
\[
10/5q_x = 10p_x \cdot 5q_{x+10} = 10p_x \cdot (1 - 5f_{x+10})
\]
\[
= 0.90 \times (1 - \frac{17}{18}) = \frac{0.90}{18} = 0.05
\]

\[(c)\]
\[
15p_x = 5f_x \cdot 10p_{x+5} \Rightarrow 10p_{x+5} = \frac{15p_x}{5f_x} = \frac{0.85}{0.90/0.95}
\]
\[
= 0.8972
\]
3. (40 points) You are given the force of mortality:

\[
\mu_x = \begin{cases} 
0.02, & 0 < x < 50 \\
0.04, & x \geq 50 
\end{cases}
\]

Calculate the following:

(a) [10 points] the probability that (30) will live another 15 years;

(b) [10 points] the probability that (65) will die before reaching age 75;

(c) [10 points] the probability that (40) will die between ages 45 and 55; and

(d) [10 points] the average lifetime of (40)

\[
\begin{align*}
\text{(a)} & \quad 15\bar{d}_{30} = e^{-0.02(15)} = e^{-0.3} \\
& \quad = 0.7408 \\
\text{(b)} & \quad 10q_{65} = 1 - 10\bar{d}_{65} \\
& \quad = 1 - e^{-0.04(10)} = 1 - e^{-0.4} = 0.629968 \\
\text{(c)} & \quad 510q_{40} = 5\bar{d}_{40} \times 10q_{45} \\
& \quad = 5\bar{d}_{40} \times (1 - 5\bar{d}_{45} \times 5\bar{d}_{50}) \\
& \quad = e^{-0.02(5)} \times (1 - e^{-0.02(5)} \times e^{-0.04(5)}) \\
& \quad = e^{-1} \times (1 - e^{-3}) = 0.2345 \\
\text{(d)} & \quad \bar{d}_{40} = E[T_{40}] = \int_0^\infty t\bar{d}_{40}dt = \int_0^{10} t\bar{d}_{40}dt + \int_{10}^{50} t\bar{d}_{40}dt \\
& \quad = \int_0^{10} e^{-0.02t}dt + \int_{10}^{50} e^{-0.02(t-10)}e^{0.04t}dt = \frac{1}{0.02}(1 - e^{-2}) + e^{-2}/0.04 = 29.53173
\end{align*}
\]