Correlated Loss Triangles for Multiple Lines of Business

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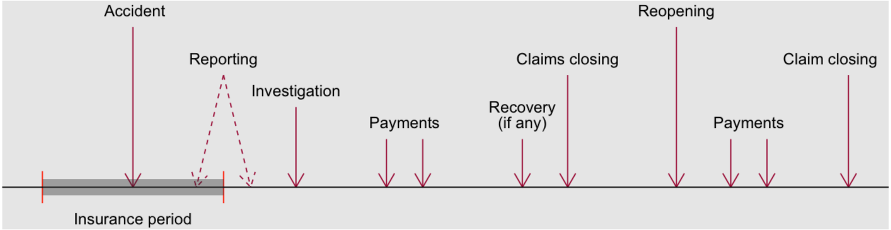
Timeline of non-life insurance claims



This diagram displays how claims may be typically processed for non-life insurance or similar products.

It illustrates why there is a need to hold loss reserves for claims, possibly already incurred, but not yet reported.

This diagram was inspired by similar graphs from Wüthrich and Merz (2008) and Frees (2010).

The importance of loss reserves

- Generally to ensure enough funds to cover losses that have yet to be paid or are expected to be paid.
- Helpful to the company for:
  - assessing its financial health
  - establishing capital needs
  - strategic planning and forecasting
  - meeting regulatory requirements
  - assessing adequacy of premiums
- When companies have several lines of business, or even different product types within a line of business, it is useful:
  - to examine the presence of possible dependencies within the structure of the company; and
  - to be able the financial effect of these dependencies.
- See G. Taylor (2000).

Run-off triangles for loss reserving

- For purposes of loss reserving:
  - widely popular to formulate the development of claims in a loss run-off triangle format;
  - such a run-off triangle gives you the observable claims for a particular accident period over the course of several periods;
  - for insurers with multiple lines, often we see that they maintain indvidual loss triangles for each line.
- Insurance companies would have an interest in both understanding the impact of each line of business to the aggregate loss reserves.
  - It has been shown that simple addition of aggregating loss reserves does not provide a very accurate picture of the total reserves needed.
  - See, for example, Ajane (1994) and Schmidt (2006).

Empirical data

- The empirical part of our investigation was motivated by:
  - the loss triangles derived from several lines of business from an insurance company;
  - data was observed over a period of ten years: 1988 to 1997;
- The different lines of business considered were:
  - Home owners/farm owners
  - Private passenger auto - liability/medical
  - Commercial multiple peril
  - Other liability - occurrence
- For each line of business, we observe the following:

Variable	Description
AccidentYear	The year that accident was occurred
DevelopmentLag	Incurral year + 1
IncurLoss	Incurred losses and allocated expenses reported at year end
CumPaidLoss	Cumulative and paid losses and allocated expenses at year end
EarnedPremN	Premiums earned at incurral year - net

The accumulation of paid losses in a run-off triangle

Home owners/Farm owners insurance											
		Development year									
Accident year	Premiums	0	1	2	3	4	5	6	7	8	9
1988	94,070	35,605	52,161	54,137	55,539	56,476	57,403	57,815	58,104	58,225	58,396
1989	95,508	44,730	63,955	65,957	69,086	67,497	69,923	70,510	70,676	70,704	
1990	92,420	36,486	50,508	53,424	55,501	56,295	56,790	57,116	57,243		
1991	101,766	48,418	64,347	68,343	69,696	70,595	70,847	71,209			
1992	112,464	96,567	68,343	135,037	136,941	137,795	138,297				
1993	128,460	61,010	80,471	84,079	85,744	86,502					
1994	143,295	78,147	95,470	100,343	103,247						
1995	150,882	67,096	83,911	87,414							
1996	121,487	75,116	90,978								
1997	32,694	10,779									

Private passenger - auto liability/medical insurance											
		Development year									
Accident year	Premiums	0	1	2	3	4	5	6	7	8	9
1988	906,236	224,230	444,587	550,263	614,499	646,555	661,208	670,101	674,655	676,514	679,176
1989	964,751	243,325	479,993	597,425	662,098	697,533	710,619	716,832	719,696	721,908	
1990	1,015,900	256,357	510,765	634,461	705,322	738,529	752,960	761,574	767,622		
1991	1,117,065	276,302	549,534	687,614	759,545	792,495	807,148	815,227			
1992	1,238,859	318,085	638,439	790,579	863,181	904,970	922,784				
1993	1,362,581	361,131	719,340	871,564	949,251	991,851					
1994	1,522,338	413,286	802,548	968,688	1049,053						
1995	1,704,342	463,972	876,510	1047,437							
1996	1,901,566	509,094	945,440								
1997	2,161,063	584,107									

Some works on loss reserving

- On single lines of business:
  - Mack (1993). *"Distribution-free calculation of the standard error of chain-ladder reserve estimates."* ASTIN Bulletin 23 (2): 213-225.
  - Taylor (2000). *"Loss Reserving: An Actuarial Perspective."* Kluwer Academic Publishers.
  - England and Verrall. (2002). *"Stochastic Claims Reserving in General Insurance."* British Actuarial Journal 8 (3): 443-518.

- On several lines of business:
  - Ajne (1994). *"Additivity of chain-ladder projections."* ASTIN Bulletin 24 (2): 313-318.
  - Braun (2004). *"The prediction error of the chain ladder method applied to correlated run-off triangles."* ASTIN Bulletin 34 (2): 399-423.
  - Schmidt, K.D. (2006). *"Optimal and additive loss reserving for dependent lines of business."* In: Casualty Actuarial Society (CAS) Forum Fall, pp. 319-351.
  - Zhang (2010). *"A general multivariate chain ladder model."* Insurance: Mathematics and Economics 46: 588-599.
  - Shi and Frees. (2011). *"Dependent loss reserving using copulas."* ASTIN Bulletin 41: 449-486.
  - Shi, Basu and Meyers. (2012). *"A Bayesian log-normal model for multivariate loss reserving."* North America Actuarial Journal 16 (1): 29-51.

Using multivariate longtudinal data framework

- We approached the problem a bit differently by borrowing ideas from multivariate longitudinal data analysis:
  - use of a random effects model to capture dynamic dependency and heterogeneity, and
  - a copula function to incorporate dependency among the response variables.
- Our response variable is a random vector of the form:
  - incremental claims, from the run-off triangles, denoted by $y_{ij,k}$, where we normalized these claims by dividing them with an exposure $\omega_{i,k}$
  - this exposure is the net premiums earned in the i-th accident year for the k-th line of business
- In effect, we used for responses "incremental loss ratios" to develop the loss run-off triangles.

ILRH	Incremental loss ratio for Home owners/farm owners
ILRP	Incremental loss ratio for Private passenger auto - liability/medical
ILRC	Incremental loss ratio for Commercial multiple peril
ILRO	Incremental loss ratio for Other liability - occurrence

Notation

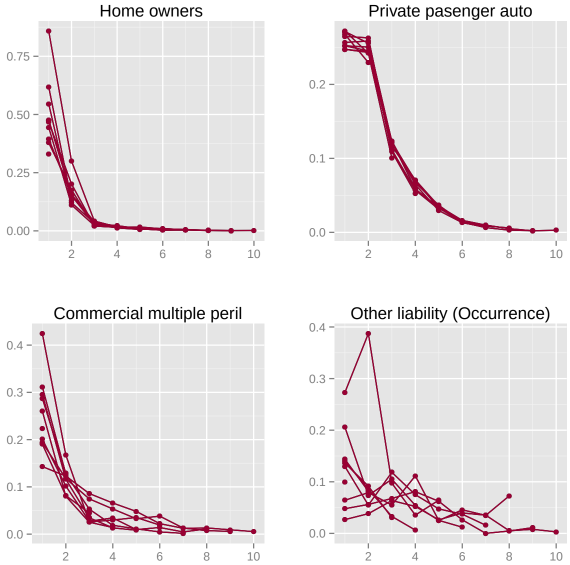
Suppose we have a set of q covariates associated with n subjects collected over T time periods for a set of m response variables.

- Let $y_{it,k}$ denote the responses from i^{th} subject in t^{th} time period on the k^{th} response. By letting $\mathbf{y_{it}} = (y_{it,1}, y_{it,2}, \ldots, y_{it,m})'$ for $t = 1, 2, \ldots, T$, we can express $\mathbf{Y_i} = (\mathbf{y_{i1}}, \mathbf{y_{i2}}, \ldots, \mathbf{y_{iT}})$.
- Covariates associated with the i^{th} subject in t^{th} time period on the k^{th} response can be expressed as $\mathbf{x_{it}} = (\mathbf{x_{it,1}}, \mathbf{x_{it,2}}, \ldots, \mathbf{x_{it,m}})$ where $\mathbf{x_{it,k}} = (x_{it1,k}, x_{it2,k}, \ldots, x_{itp,k})$ for $k = 1, 2, ...m$.
- We use α_{ik} to represent the random effects component corresponding to the i^{th} subject from the k^{th} response variable.
- $G\left(\alpha_{ik}\right)$ represents the pre-specified distribution function of random effect α_{ik}.
- For our purpose, subject i is accident year.

Key features of our approach

- Obviously, the extension from univariate to multivariate longitudinal analysis.
- Types of dependencies captured:
  - the dependence structure of the response using copulas - provides flexibility
  - the intertemporal dependence within subjects and unobservable subject-specific heterogeneity captured through the random effects component - provides tractability
- The marginal distribution models:
  - any family of flexible enough distributions can be used
  - choose family so that covariate information can be easily incorporated
- Other key features worth noting:
  - the parametric model specification provides flexibility for inference e.g. MLE for estimation
  - model construction can accommodate both balanced and unbalanced data - an important feature for longitudinal data

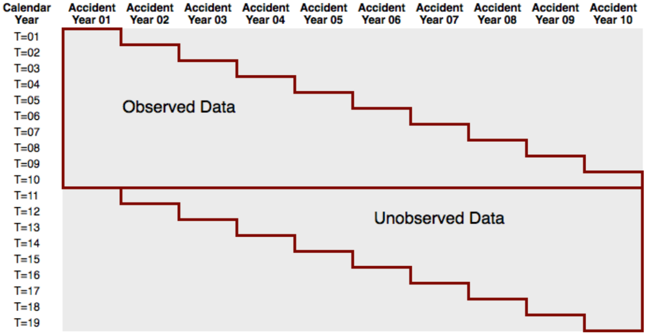
Incremental loss ratio of four different insurance lines										
	Development year									
Accident year	0	1	2	3	4	5	6	7	8	9
1988	0.3785	0.1760	0.0210	0.0149	0.0100	0.0099	0.0044	0.0031	0.0013	0.0018
	0.2474	0.2432	0.1166	0.0709	0.0354	0.0162	0.0098	0.0050	0.0021	0.0029
	0.1430	0.1229	0.0859	0.0659	0.0481	0.0222	0.0124	0.0131	0.0092	0.0054
	0.0267	0.0382	0.0636	0.0522	0.0263	0.0396	0.0353	0.0051	0.0079	0.0030
1989	0.4683	0.2013	0.0210	0.0161	0.0166	0.0088	0.0061	0.0017	0.0003	
	0.2522	0.2453	0.1217	0.0670	0.0367	0.0136	0.0064	0.0030	0.0023	
	0.1939	0.1177	0.0746	0.0534	0.0326	0.0381	0.0129	0.0074	0.0059	
	0.0478	0.0561	0.0677	0.0809	0.0622	0.0263	0.0001	0.0046	0.0112	
1990	0.3948	0.1517	0.0316	0.0225	0.0086	0.0054	0.0035	0.0014		
	0.2523	0.2504	0.1218	0.0698	0.0327	0.0142	0.0085	0.0060		
	0.2009	0.1017	0.0274	0.0343	0.0097	0.0140	0.0047	0.0111		
	0.0642	0.0790	0.0549	0.1115	0.0248	0.0457	0.0349	0.0727		
1991	0.4758	0.1565	0.0393	0.0133	0.0088	0.0025	0.0036			
	0.2473	0.2446	0.1236	0.0644	0.0295	0.0131	0.0072			
	0.3115	0.1270	0.0528	0.0187	0.0112	0.0047	0.0017			
	0.1297	0.0548	0.1184	0.0750	0.0474	0.0372	0.0158			
1992	0.8586	0.2998	0.0422	0.0169	0.0076	0.0045				
	0.2568	0.2586	0.1228	0.0586	0.0337	0.0144				
	0.4245	0.1673	0.0257	0.0298	0.0352	0.0192				
	0.2061	0.0736	0.1049	0.0542	0.0252	0.0120				
1993	0.4749	0.1515	0.0281	0.0130	0.0059					
	0.2650	0.2629	0.1117	0.0570	0.0313					
	0.2959	0.1162	0.0346	0.0133	0.0083					
	0.2730	0.3873	0.0973	0.0354	0.0644					
1994	0.5454	0.1209	0.0340	0.0203						
	0.2715	0.2557	0.1091	0.0528						
	0.2602	0.0802	0.0253	0.0140						
	0.1398	0.0852	0.0329	0.0064						
1995	0.4447	0.1114	0.0232							
	0.2722	0.2421	0.1003							
	0.1905	0.0815	0.0448							
	0.1365	0.0915	0.0303							
1996	0.6183	0.1306								
	0.2677	0.2295								
	0.2874	0.1289								
	0.1436	0.0817								
1997	0.3297									
	0.2703									
	0.2231									
	0.0999									



x-axis: Development lag and y-axis: Incremental loss ratio

Calibrating the model

- We transformed the loss triangle data into a longitudinal framework.
- Calendar year represents the time and each accident year represents a subject.
- This framework allows us to perform multivariate longitudinal data analysis.
- We only have one covariate: the development year.



The following set up allows us to use development year 1 as the base factor.

	D2	D3	D4	D5	D6	D7	D8	D9	D10
Dev.Year 1	0	0	0	0	0	0	0	0	0
Dev.Year 2	1	0	0	0	0	0	0	0	0
Dev.Year 3	0	1	0	0	0	0	0	0	0
Dev.Year 4	0	0	1	0	0	0	0	0	0
Dev.Year 5	0	0	0	1	0	0	0	0	0
Dev.Year 6	0	0	0	0	1	0	0	0	0
Dev.Year 7	0	0	0	0	0	1	0	0	0
Dev.Year 8	0	0	0	0	0	0	1	0	0
Dev.Year 9	0	0	0	0	0	0	0	1	0
Dev.Year 10	0	0	0	0	0	0	0	0	1

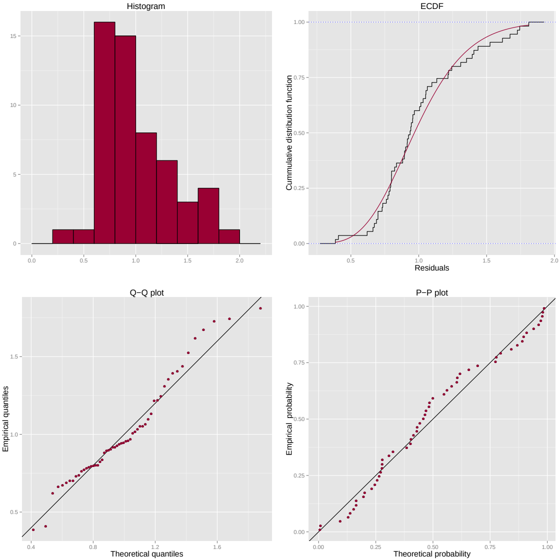
Fitting the marginals

Marginals	**Density** $f(y)$	**Covariates**	**Residuals** R_i	**Line of business**
Gamma	$\frac{1}{\Gamma(\nu)y}\left(\frac{y\nu}{\mu}\right)^{\nu} e^{(-y\nu/\mu)}$	$\log \mu_i(\mathbf{x}) = \alpha_i + \beta'\mathbf{x}$	$\frac{Y_i}{\mu_i(\mathbf{x})}$	ILRH
Log-normal	$\frac{1}{\sigma\sqrt{2\pi}y} \exp\left[-\frac{(\log(y)-\mu)^2}{2\sigma^2}\right]$	$\mu_i(\mathbf{x}) = \alpha_i + \beta'\mathbf{x}$	$\frac{\log(Y_i) - \mu_i(\mathbf{x})}{\sigma}$	ILRP, ILRC
Weibull	$\frac{\kappa}{\lambda}\left(\frac{y}{\lambda}\right)^{\kappa-1} e^{-(y/\kappa)^{\kappa}}$	$\log \lambda_i(\mathbf{x}) = \alpha_i + \beta'\mathbf{x}$	$\frac{Y_i}{\lambda_i(x)}$	ILRO

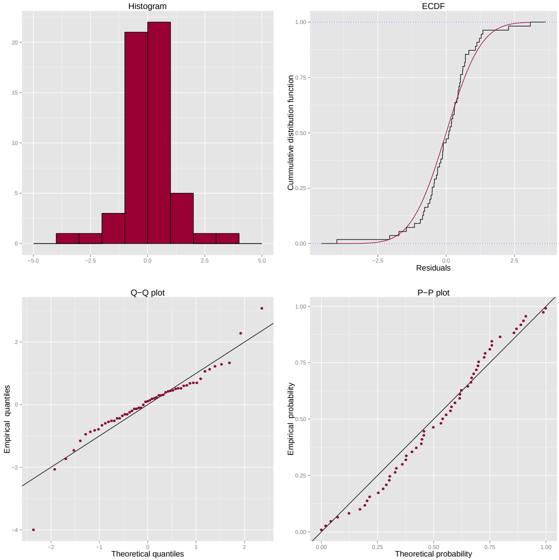
Fitted models for the various marginals

	Lines of Business											
	ILRH			**ILRP**			**ILRC**			**ILRO**		
	Gamma distribution			**Log-normal distribution**			**Log-normal distribution**			**Weibull distribution**		
Parameter	**Estimate**	**Std Error**	**p-val**	**Estimate**	**Std Error**	**p-val**	**Estimate**	**Std Error**	**p-val**	**Estimate**	**Std Error**	**p-val**
Covariates												
Dev.Year 1	-	-	-	-	-	-	-	-	-	-	-	-
Dev.Year 2	-1.0997	0.1435	0.0000	-0.0479	0.0451	0.2936	-0.7626	0.1947	0.0003	-0.1771	0.2835	0.5356
Dev.Year 3	-2.8077	0.1480	0.0000	-0.8103	0.0466	0.0000	-1.7508	0.2023	0.0000	-0.5871	0.2801	0.0420
Dev.Year 4	-3.3975	0.1540	0.0000	-1.4253	0.0486	0.0000	-2.1746	0.2113	0.0000	-0.7229	0.2983	0.0196
Dev.Year 5	-3.9561	0.1621	0.0000	-2.0632	0.0511	0.0000	-2.5806	0.2229	0.0000	-1.1480	0.3095	0.0006
Dev.Year 6	-4.3969	0.1760	0.0000	-2.9075	0.0544	0.0000	-2.8066	0.2383	0.0000	-1.3550	0.3331	0.0002
Dev.Year 7	-4.7331	0.1859	0.0000	-3.5022	0.0589	0.0000	-3.7751	0.2579	0.0000	-1.6428	0.3670	0.0001
Dev.Year 8	-5.4997	0.2112	0.0000	-4.0707	0.0663	0.0000	-3.3521	0.2900	0.0000	-1.3261	0.4192	0.0029
Dev.Year 9	-6.4610	0.2564	0.0000	-4.7769	0.0772	0.0000	-3.8281	0.3451	0.0000	-2.5145	0.4739	0.0000
Dev.Year 10	-5.6530	0.3411	0.0000	-4.5198	0.1074	0.0000	-4.1785	0.4649	0.0000	-3.6613	0.6379	0.0000
Intercept	-0.6969	0.1002	0.0000	-1.3467	0.0320	0.0000	-1.4181	0.1573	0.0000	-1.9998	0.1980	0.0000
Marginals												
ν	6.3380	1.2835	0.0000	-	-	-	-	-	-	-	-	-
σ	-	-	-	0.0980	0.0100	0.0000	0.4207	0.0436	0.0000	-	-	-
κ	-	-	-	-	-	-	-	-	-	1.7299	0.2081	0.0000
Random effect												
σ_α	0.0561	0.0967	0.5646	0.2651	0.0870	0.0039	0.2651	0.0870	0.0039	0.2042	0.1158	0.0850

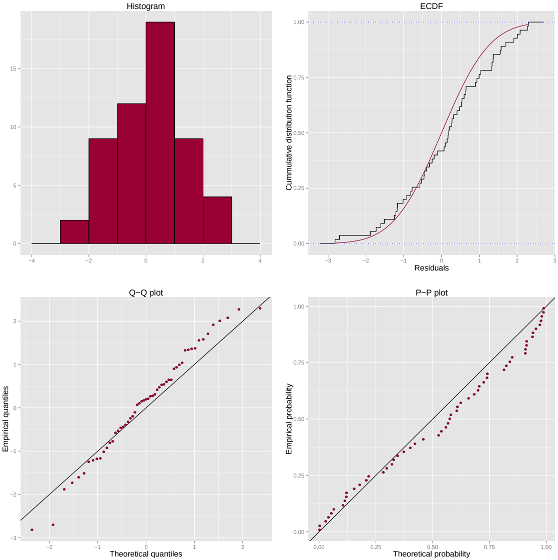
Graphical diagnostics - ILRH



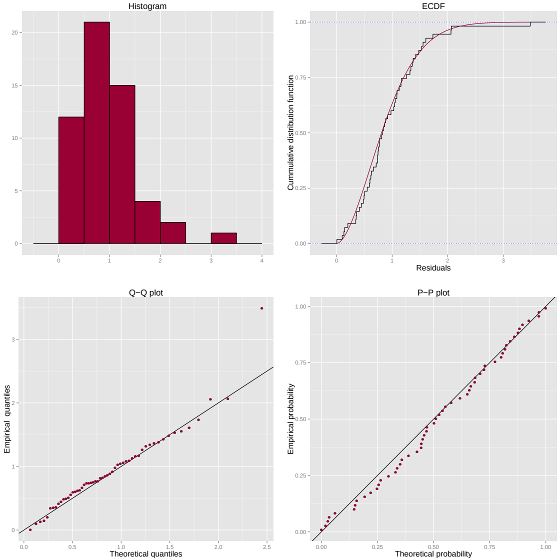
Graphical diagnostics - ILRP



Graphical diagnostics - ILRC



Graphical diagnostics - ILRO



Investigating the time dependence

We may be able to capture the serial correlation within each response variable,k, by incorporating subject-specific random effects, α_{ik}.

Just using basic statistics:

- Correlation between y_{i1k} and y_{i2k}:

$$\mathsf{Corr}(y_{i1k}, y_{i2k}) = \frac{\mathsf{Cov}(y_{i1k}, y_{i2k})}{\sqrt{\mathsf{Var}(y_{i1k})\mathsf{Var}(y_{i2k})}}$$

  where we have

$$\mathsf{Cov}(y_{i1k}, y_{i2k}) = \mathsf{E}(\mathsf{Cov}(y_{i1k}, y_{i2k})|\alpha_{ik}) + \mathsf{Cov}(\mathsf{E}(y_{i1k}|\alpha_{ik}), \mathsf{E}(y_{i2k}|\alpha_{ik}))$$

- Under the conditional independence:

$$\mathsf{Cov}(y_{i1k}, y_{i2k}) = \mathsf{Cov}(\mathsf{E}(y_{i1k}|\alpha_{ik}), \mathsf{E}(y_{i2k}|\alpha_{ik}))$$

- $\mathsf{Var}(y_{ijk})$ can be expressed as:

$$\mathsf{Var}(y_{ijk}) = \mathsf{E}(\mathsf{Var}(y_{ijk}|\alpha_{ik})) + \mathsf{Var}(\mathsf{E}(y_{ijk}|\alpha_{ik}))$$

Serial correlations of the response variables

ILRH

	CY 6	CY 7	CY 8	CY 9
CY 10	0.788	0.825	0.845	0.888
CY 9	0.817	0.841	0.888	
CY 8	0.839	0.883		
CY 7	0.879			

ILRP

	CY 6	CY 7	CY 8	CY 9
CY 10	0.855	0.912	0.928	0.909
CY 9	0.906	0.923	0.904	
CY 8	0.914	0.896		
CY 7	0.885			

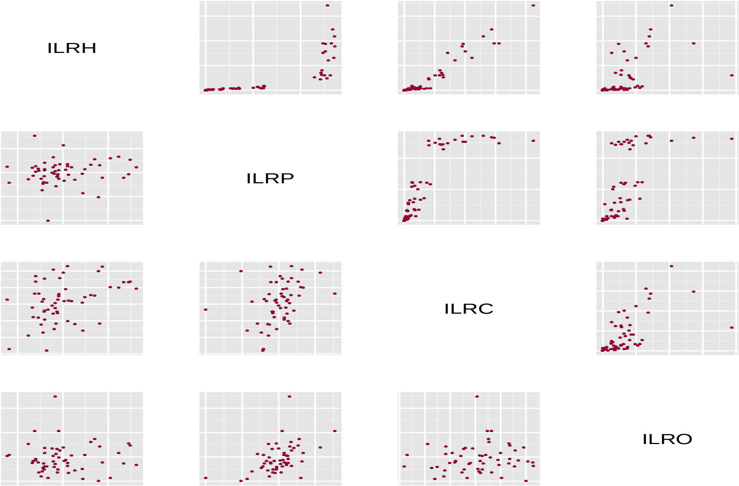
ILRC

	CY 6	CY 7	CY 8	CY 9
CY 10	0.630	0.669	0.703	0.773
CY 9	0.642	0.692	0.764	
CY 8	0.667	0.757		
CY 7	0.744			

ILRO

	CY 6	CY 7	CY 8	CY 9
CY 10	0.110	0.123	0.108	0.161
CY 9	0.081	0.101	0.099	
CY 8	0.069	0.095		
CY 7	0.099			

Correlation matrix



Note: The upper part gives the loss ratios from raw data, while the lower part as the residuals after fitting marginals.

Estimates for various copula models

- Family of Archimedean copulas:

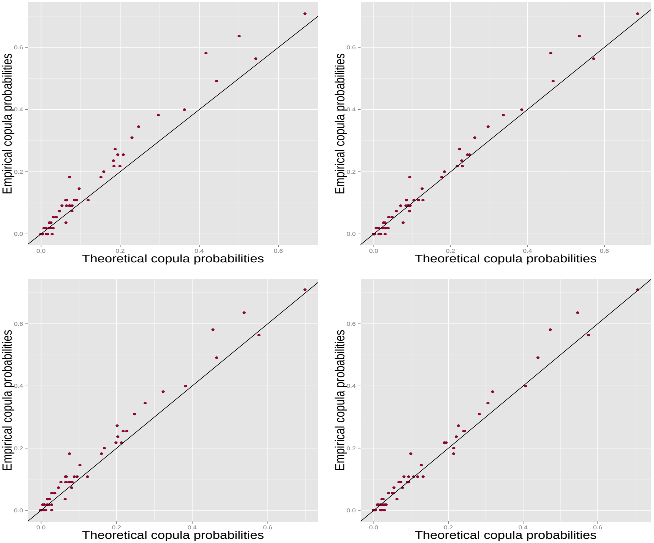
Copulas	Parameter estimates	Standard error	p-value	AIC	BIC
Clayton	0.1499	0.0566	0.0106	447.8156	449.8229
Frank	1.4907	0.4284	0.0010	441.6951	443.7024
Gumbel	1.1335	0.0526	0.0141	447.8317	449.8390

- Family of elliptical copulas

	Normal copula			t-copula		
Parameter	**Estimate**	**Std Error**	**p-val**	**Estimate**	**Std Error**	**p-val**
r_{12}	0.0746	0.1289	0.5656	0.0797	0.1326	0.5506
r_{13}	0.3472	0.0963	0.0007	0.3429	0.0984	0.0011
r_{14}	-0.0563	0.1182	0.6362	-0.0439	0.1227	0.7219
r_{23}	0.3126	0.0969	0.0022	0.3201	0.0990	0.0022
r_{24}	0.5309	0.0785	0.0000	0.5290	0.0816	0.0000
r_{34}	0.0282	0.1005	0.7801	0.0404	0.1041	0.6998
df	-	-	-	75.9600	80.5460	0.3504
AIC		426.9003			429.5532	
BIC		438.9443			443.6046	

Here, r_{ij} represent the correlation between i^{th} and j^{th} insurance lines.
1: ILRH, 2: ILRP, 3: ILRC and 4: ILRO

Using pp-plots for copula validation



From top left to bottom right: Clayton, Frank, Gumbel and Normal

Reserve estimates by accident year - Normal copula

- By line of business

Accident Year	ILRH Lower Bound	ILRH Predicted Value	ILRH Upper Bound	ILRP Lower Bound	ILRP Predicted Value	ILRP Upper Bound	ILRC Lower Bound	ILRC Predicted Value	ILRC Upper Bound	ILRO Lower Bound	ILRO Predicted Value	ILRO Upper Bound
1989	91	167	263	2,316	2,749	3,232	93	237	474	18	92	205
1990	127	234	368	4,324	5,132	6,036	52	134	269	28	145	323
1991	252	466	732	8,945	10,629	12,505	94	241	480	92	475	1,056
1992	549	1,010	1,578	18,170	21,552	25,315	126	309	603	160	823	1,828
1993	1,053	1,945	3,054	36,334	43,180	50,754	155	400	794	343	1,764	3,903
1994	1,917	3,541	5,557	83,277	98,835	116,148	231	593	1,182	1,992	10,294	22,857
1995	3,388	6,248	9,803	183,253	217,781	255,976	366	942	1,884	3,211	16,662	37,105
1996	4,703	8,701	13,674	390,883	464,283	545,702	545	1,399	2,793	4,737	24,486	54,173
1997	4,222	7,783	12,180	898,051	1,066,427	1,253,355	1,129	2,913	5,825	7,338	37,960	84,451

- Combined lines of business

Calendar Year	Avg - 2×StdDev	Lower Bound	Predicted Value	Upper Bound	Avg + 2×StdDev
1989	2,458	2,651	3,245	3,938	4,032
1990	4,401	4,695	5,646	6,732	6,891
1991	9,103	9,749	11,811	14,183	14,520
1992	18,475	19,692	23,693	28,254	28,911
1993	36,732	39,167	47,289	56,537	57,845
1994	83,316	91,068	113,262	139,929	143,209
1995	181,821	196,740	241,633	294,272	301,445
1996	382,850	410,864	498,869	600,658	614,888
1997	866,081	924,983	1,115,084	1,331,428	1,364,087

Reserve estimates by accident year - Frank copula

- By line of business

Accident Year	ILRH Lower Bound	ILRH Predicted Value	ILRH Upper Bound	ILRP Lower Bound	ILRP Predicted Value	ILRP Upper Bound	ILRC Lower Bound	ILRC Predicted Value	ILRC Upper Bound	ILRO Lower Bound	ILRO Predicted Value	ILRO Upper Bound
1989	91	167	263	2,315	2,748	3,230	91	237	473	17	92	203
1990	128	234	368	4,323	5,131	6,030	52	134	268	28	145	322
1991	254	466	733	8,953	10,626	12,489	93	241	482	91	474	1,052
1992	551	1,010	1,588	18,157	21,546	25,323	120	309	619	156	821	1,822
1993	1,061	1,945	3,056	36,355	43,157	50,734	154	399	798	336	1,759	3,896
1994	1,931	3,541	5,563	83,228	98,794	116,129	230	593	1,187	1,956	10,270	22,750
1995	3,411	6,253	9,830	183,425	217,759	255,954	365	945	1,887	3,192	16,661	36,954
1996	4,745	8,701	13,671	391,010	464,106	545,569	542	1,399	2,799	4,662	24,450	54,297
1997	4,251	7,785	12,237	897,944	1,066,094	1,253,066	1,124	2,915	5,842	7,234	37,878	83,976

- Combined lines of business

Accident Year	Avg - 2×StdDev	Lower Bound	Predicted Value	Upper Bound	Avg + 2×StdDev
1989	2,474	2,670	3,243	3,916	4,012
1990	4,426	4,712	5,644	6,700	6,863
1991	9,198	9,823	11,808	14,071	14,417
1992	18,552	19,755	23,687	28,127	28,821
1993	37,021	39,430	47,259	56,117	57,497
1994	85,213	92,386	113,197	137,827	141,181
1995	185,643	199,355	241,617	290,439	297,590
1996	388,967	414,736	498,655	593,709	608,344
1997	877,550	931,686	1,114,673	1,318,771	1,351,797

Model comparison

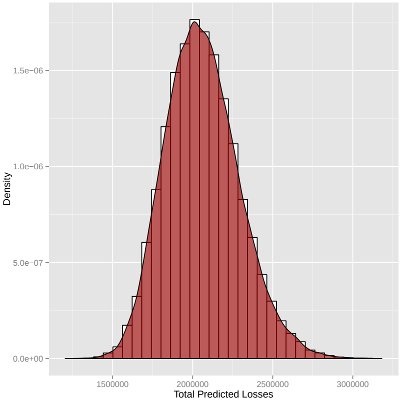
We compared the results from the above models with the multivariate chain ladder method.

Accident year	Multivariate chain ladder	Normal copula	Frank copula	Independence model
1989	3,463	3,245	3,243	3,464
1990	5,858	5,646	5,644	5,858
1991	11,978	11,811	11,808	11,978
1992	25,938	23,693	23,687	25,713
1993	50,797	47,289	47,259	50,817
1994	112,001	113,262	113,197	112,096
1995	234,878	241,633	241,617	232,873
1996	483,958	498,869	498,655	481,088
1997	1,129,869	1,115,084	1,114,673	1,124,843
Total Reserve	2,058,740	2,060,532	2,059,783	2,048,730

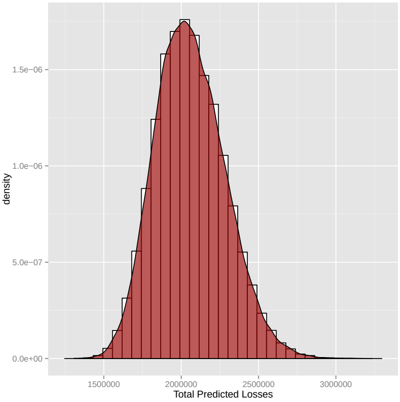
The predictive distributions

Total Reserves

Normal copula

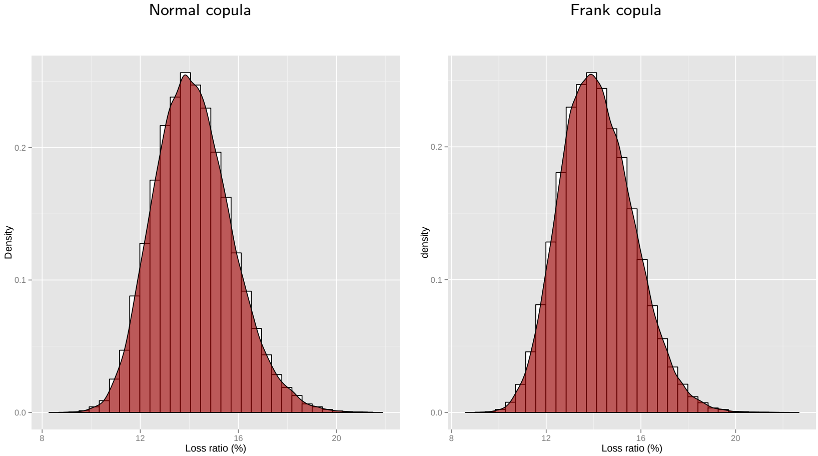


Frank copula

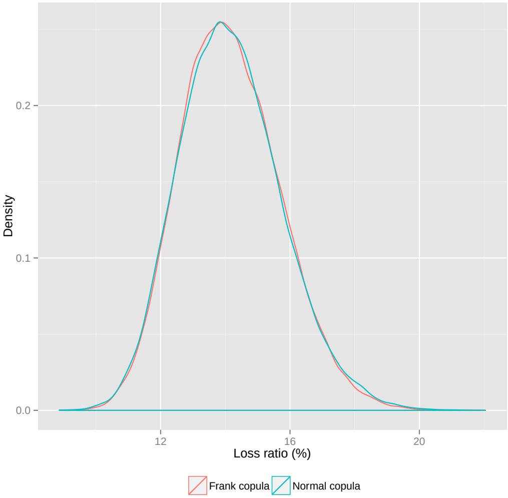


- continued

Total reserves per dollar premium exposure



Model comparison



Concluding remarks

- This work is still on its premature stage.
- Our work intends to provide addition to the growing literature on modeling dependence on run-off triangles for multiple lines of business:
  - time dependence
  - dependence across the various lines of business
- We wish to exploit the use of multivariate longitudinal data analysis.
  - There is a growing literature on the statistical methods for multivariate longitudinal data.
  - There is also a growing literature on the use of such methodology in disciplines such as biostatistics.
- Our future research work includes many things including:
  - improving model selection criteria; and
  - understanding the predictions arising from various models by doing some sensitivity analysis.