2. Let $p$ be an integer other than $0, \pm 1$. Prove that $p$ is prime if and only if for each $a \in \mathbb{Z}$ either $\gcd(a, p) = 1$ or $p \mid a$.

4. Let $p$ be an integer other than $0, \pm 1$ with the following property: Whenever $b$ and $c$ are integers such that $p \mid bc$, then $p \mid b$ or $p \mid c$. Prove that $p$ is prime.

   **Hint:** If $d$ is a divisor of $p$, say $p = dt$ for some $d, t \in \mathbb{Z}$, then $p \mid d$ or $p \mid t$. Show that this implies $d = \pm p$ or $d = \pm 1$.

10. Assume $a, b, c, d \in \mathbb{Z}$. Prove or disprove each of the following statements.

   (a) If $p$ is prime and $p \mid (a^2 + b^2)$ and $p \mid (c^2 + d^2)$, then $a^2 - c^2$.

   (b) If $p$ is prime and $p \mid (a^2 + b^2)$ and $p \mid (c^2 + d^2)$, then $a^2 + c^2$.

   (c) If $p$ is prime and $p \mid a$ and $p \mid (a^2 + b^2)$, then $b$.

25. (Bonus - you can use this problem to substitute two problems of your choice from group A assigned today and Monday, 9/8.) Prove or disprove: If $n$ is an integer and $n > 2$, then there exists a prime $p$ such that $n < p < n!$. 