Math 299 **Lecture 5: Sections 2.6 and 2.7.**
Biconditional, Converse, Tautologies and Contradictions

Converse

The statement $B \Rightarrow A$ is the converse of the statement $A \Rightarrow B$.

Give an example for each of the following cases:

- The statement is true and its converse is false.

- The statement is false and its converse is true.

- Both the statement and its converse are true.

- Both the statement and its converse are false.

Relationship between a statement, its inverse, its converse and its contrapositive.

If-and-only-if (biconditional) statements

If both A $\Rightarrow$ B and B $\Rightarrow$ A are true, we write A $\Leftrightarrow$ B

- "A and B are equivalent"
- "A holds if and only if B holds"
- Abbreviation: "A iff B"
- "A is necessary and sufficient for B"

Truth table for $A \iff B$

A	B	$A \Rightarrow B$	$B \Rightarrow A$	$(A \Rightarrow B) \wedge (B \Rightarrow A)$	$A \iff B$
T	T				
T	F				
F	T				
F	F				

Proving if-and-only-if statements

- Let $x, y \in \mathbb{R}$. Then $(x+y)^2 = x^2 + y^2$ if and only if $xy = 0$.

- The integer n is odd if and only if $n+1$ is even.

- The natural number n is odd if and only if n^2 is odd.

- A rectangle with perimeter $4a$ is a square if and only if its area is a^2.

- *(Challenge.)* Define the Euclidean norm of $\mathbf{x} = (x_1, x_2, ..., x_n) \in \mathbb{R}^n$ by $||\mathbf{x}|| = \sqrt{x_1^2 + ... + x_n^2}$. Then $||\mathbf{x}|| = 0$ if and only if $(x_1, x_2, ..., x_n) = (0, 0, ..., 0)$.

Tautology

Definition: Tautology is a compound statement that is always true, regardless of the truth value of the individual statements.

Examples:

1. $P \iff P$
2. $Q \vee (\sim Q)$

Prove that $(P \vee Q) \vee (P \vee (\sim Q))$ is a tautology.

P	Q	$\sim Q$	$P \vee Q$	$P \vee (\sim Q)$	$(P \vee Q) \vee (P \vee (\sim Q))$
T	T				
T	F				
F	T				
F	F				

Contradiction

Definition: Contradiction is a compound statement that is always false, regardless of the truth value of the individual statements.

Examples:

1. $P \iff (\sim P)$
2. $Q \wedge (\sim Q)$

Theorem: A statement S is a tautology if and only if its negation is a contradiction.