1. Let $n \in \mathbb{N}$ and $I = \{1, 2, \ldots, n\}$. For $i \in I$, define $A_i = [(i-1)/n, i/n]$. Identify each of the following sets by writing it as an interval or a union of two intervals.
   (a) $\bigcup_{i \in I} A_i$
   (b) $\bigcap_{i \in I} \overline{A_i}$
2. Consider the set $S = \{\emptyset, \square\}$.
   (a) List the elements of $\mathcal{P}(S)$.
   (b) List the elements of $\mathcal{P}(\mathcal{P}(S))$.
   (c) Find a partition of $\mathcal{P}(\mathcal{P}(S))$ into 3 sets.
   (d) Is it possible to find a partition of S into 3 sets? Explain.
3. Prove the statements appearing in (a)-(b), and answer the prompt in (c)-(d). The symbol $\equiv$ denotes *congruence* modulo n, where $n \in \mathbb{Z}$ such that $n \geq 2$.
   (a) For all $a, b \in \mathbb{Z}$, if $a \equiv b$, then $b \equiv a$.
   (b) For all sets $a, b, c \in \mathbb{Z}$, if $a \equiv b$ and $b \equiv c$, then $a \equiv c$.
   (c) State the negation of each of the statements (a)-(b) above. Determine if the negation is true or false. Provide a counterexample for any false statement.
   (d) Let $a, b, c \in \mathbb{Z}$, and consider the conditional statement

       P: If $a \equiv b$ and $b \equiv c$, then $a \equiv c$.

       State the inverse, contrapositve and converse of statement P. Determine whether each of these is true or false.
4. Negate the following.
   (a) $\forall n \in \mathbb{Z}, \quad \exists m \in \mathbb{Z}$ such that $m \cdot n = 1$.
   (b) $\exists x \in \mathbb{Q}$ such that $\forall y \in Q, \quad x \cdot y = y$.

   Rewrite the statements in (a) and (b) without the use of the symbols $\forall, \exists$, and state whether each is a true or a false statement. If it is a true statement, prove it. If it is a false statement, provide a counterexample.
5. Construct a truth table to show that the contrapositive of $A \Rightarrow B$ is equivalent to $A \Rightarrow B$.
6. Prove the following statement.
   $$\forall a \in \mathbb{R}\, \exists! x \in \mathbb{R}, \text{ such that } 3x - 1 = a.$$
7. Let E denote the set of even integers, $x \in \mathbb{Z}$, and $A(x)$ be the following open sentence.
   $$A(x): \text{“}x \in E \Rightarrow \exists k \in \mathbb{Z} \text{ such that } x = 2k\text{”}$$
   (a) Write the inverse of $A(x)$.
   (b) Write the converse of $A(x)$.
   (c) Write the contrapositive of $A(x)$.
   (d) Is $A(x)$ true for all $x \in \mathbb{Z}$? What about its converse? In this case, how would you restate it using *necessary/ sufficient/ necessary and sufficient*?
8. Let $A = \{x \in \mathbb{Z} | x = 6k, k \in \mathbb{Z}\}$,
   $B = \{x \in \mathbb{Z} | x = 2k, k \in \mathbb{Z}\}$,
   $C = \{x \in \mathbb{Z} | x = 3k, k \in \mathbb{Z}\}$. Prove the following statement.
   $$x \in A \iff (\exists y \in B \text{ and } \exists z \in C \text{ such that } x = yz)$$
9. Construct a truth table to show that $A \Rightarrow B$ is equivalent to the statement: not(A) or B.
10. Let $a, b, c \in \mathbb{R}$, and consider the following open sentence:

    $P(a, b, c)$: A necessary condition for the equation $ax^2 + bx + c = 0$ to have a solution is: $a \neq 0$ and $b^2 - 4ac \geq 0$.

    (a) Rephrase $P(a, b, c)$ as an if-then implication; explicitly write all relevant quantifiers.
    (b) Write the contrapositive.
    (c) Write the converse.
    (d) Write the inverse.
    (e) Write the negation of $P(a, b, c)$ (simplified by moving the *not* as far into the statement as possible).
    (f) Which of the above statements (a)-(d) are equivalent to each other (for all $a, b, c \in \mathbb{R}$)?
    (g) The statement '$\forall a, b, c \in \mathbb{R}$, $P(a, b, c)$' is false. Disprove it (prove the negation).
11. Let $x, y \in \mathbb{R}$. Prove that $(x + y)^2 = x^2 + y^2$ if and only if $xy = 0$.
12. Let $n \in \mathbb{Z}$. Prove that n is odd if and only if $n + 7$ is even.
13. Let $n \in \mathbb{Z}$. Prove that n is odd if and only if n^2 is odd.
14. Let $a \in (0, \infty)$. Prove that a rectangle with perimeter $4a$ is a square if and only if its area is a^2.

15. Define the *Euclidean norm* of $\mathbf{x} = (x_1, x_2, ..., x_n) \in \mathbb{R}^n$ by $||\mathbf{x}|| = \sqrt{x_1^2 + ... + x_n^2}$. Prove that $||\mathbf{x}|| = 0$ if and only if $(x_1, x_2, ..., x_n) = (0, 0, ..., 0)$.

16. Let $x \in \mathbb{Z}$.

    (a) Prove that $x^2 + x$ is even.

    (b) Assume $x \neq 0$. Prove that $(x^2 + x)/2$ is divisible by x if and only if x is odd.

    (c) Assume $x + 1 \neq 0$. Prove that $(x^2 + x)/2$ is divisible by $x + 1$ if and only if x is even.

17. Show that if $x^2 - 3x + 2 < 0$, then $1 < x < 2$.

18. Let $a, b, c, d \in \mathbb{Z}$ with a and b nonzero. Prove that if $ab \nmid cd$, then $a \nmid c$ or $b \nmid d$.

19. Prove that if a positive integer is divisible by 3 then the sum if its digits is divisible by 3.

20. Prove that for any two sets A and B, $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$.

21. Prove that for any two sets A and B, $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$.

22. Prove that for any sets A, B and C, $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

23. Prove that if $n|a$ then $n|a + b \Leftrightarrow n|b$