Math 299 **Read, Understand, and Write mathematical statements** **Jan. 6, 2014**

Goal: Learn what a rigorous logical arguement is and how to write one

This is a writing course

Two reasons to focus on writing

1. It helps you understand better.

   If you can't explain it, you don't understand it.

2. Math is a human social endeavor and to be a mathematician you must learn to communicate about math.

   (a) Good for teaching

   (b) Good for research

   (c) Good for scoring well on HW/Quizzes/Exams in ALL proof based math classes.

Writing Guidelines

A math proof is, before anything else, an essay in the language in which it is written.

In particular:

1. Use complete sentances
2. Use punctuation - even in equations
3. Explain what you are doing
4. Justify your assertions

Example

Proof that the product of two evens is even:

$$2m \cdot 2n = 4mn = 2(2mn)$$

List 3-5 things that you felt could be improved on in the last example:

1.
2.
3.
4.
5.

Better Revised Example

Prove that the product of two evens is even.

Proof. Take two even numbers a & b and consider ab. By the definition of even, $a = 2m$ for some $m \in \mathbb{Z}$. Likewise, $b = 2n$ for some $n \in \mathbb{Z}$
Therefore,
$$ab = (2m)(2n) = (m2)(2n) = m4n = 4mn = 2(2mn)$$
By the definition of even, ab is even.

Notice that this is still not great because we have not introduced the symbols $\mathbb{Z}$ or $\in$ yet. In addition the ampersand does not simplify things. Also the commutativity of reals should be well understood. This should be written simpler. This will violate rules later on and will yeild a final revised edition.

Be smart about writing math

1. Equal means equal (and nothing else)

   Denotes both objects are exactly the same
   Ex of abuse: Find the derivative of x^3
   Solution: $x^3 = 3x^2$

2. Don't draw arrows everywhere
   Why would this occur?

   (a) Didn't leave enough room

   (b) Need to refer to something previously written

   Give it a name instead

   In the case of **(a)** this is just laziness. In this class we will expect EXCELLENT work. This often involves writing things down (*especially HWs*) multiple times.

   A good rule of thumb to follow is writing 3 copies of each HW.

   (1) Solving the problem

   (2) First writing draft

   (3) Final draft

   And between the first and final draft you should make sure to:

   (2.1) Read your solution, looking for errors: accuracy, typos, grammar.

   (2.2) Reflect on your work

   It is important to take pride in your HW.

3. Keep things as simple as possible (and no simpler)
4. Use symbols when they simplify (but only then).
   π, $\in$, $\mathbb{Z}$, etc.
   Make sure you define your own symbols BEFORE using them
5. Do not start a sentance with a symbol.

Final Revised Example

Prove that the product of two evens is even.

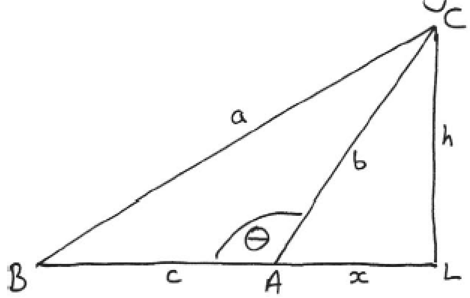
Proof. Take two even numbers a and b and consider ab. By the definition of even, $a = 2m$ for some integer m. Likewise, $b = 2n$ for some integer n
Therefore,

$$ab = (2m)(2n) = 4mn = 2(2mn)$$

By the definition of even, ab is even.

Exercise 1. Flesh out a mathematically "correct" proof of the **Law of Cosines** to make it readable.

Geometry Examples 1



Θ = obtuse angle

$\triangle CBL$

$a^2 = (c+x)^2 + h^2$

$a^2 = c^2 + 2cx + \underbrace{h^2 + x^2}_{b^2}$

$\triangle CLA$

$b^2 = h^2 + x^2$

$a^2 = b^2 + c^2 + 2cx$

In $\triangle CLA$ $\quad \dfrac{x}{b} = \cos(180 - \Theta)$

$\Rightarrow x = -b\cos\Theta$ Sub into

$a^2 = b^2 + c^2 - 2cb\cos\Theta$

Example. A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a fence. With 800m of fence available, what shape of rectangle will enlcose the largest area?

Solution A.

$$\text{Area} = ab$$

$$2a + b = 800$$

$$b = 800 - 2a$$

$$f(a) = a(800 - 2a)$$

$$f'(a) = 800 - 4a = 0 \implies a = 200, b = 400$$

Solution B. (Improved with explanations) Denote the length and width of the rectangular plot by a and b. The area is given by:

$$\text{Area} = ab.$$

Without loss of generality, we can assume that the river runs along the width b of the rectangle, and the 800m of fence runs along sides of length a, b, a; hence:

$$2a + b = 800.$$

Solving the above equation for b:

$$b = 800 - 2a.$$

Then the area as a function of the length is:

$$f(a) = a(800 - 2a),$$

where f has domain $[0, 400]$, since a valid length has $a \geq 0$, and $b \geq 0$ whenever $a \leq 400$.

Recalling the method to find the absolute maximum of $f(a)$ in its domain $a \in [0, 400]$:

1. Find the critical points in the domain, where the derivative $f'(a)$ equals zero or is undefined. We have: $f'(a) = 800 - 4a$, which is defined for all a, and $f'(a) = 0$ only for $a = 200$.
2. Evaluate $f(a)$ at the critical points: $f(200) = (200)(400) = 80000$.
3. Evaluate $f(a)$ at the endpoints of the domain: $f(0) = f(400) = 0$.
4. The absolute maximum is the largest value $f(a)$ from steps 2 and 3: that is, $f(200) = 80000$.

Thus, the largest area that can be enclosed is 80,000m^2, achieved when the plot has length $a = 200$m and width $b = 400$m.