Partitions of Sets

1. If the intersection of two sets is the empty set, then the two sets are disjoint.

2. Let S be a collection of subsets of a set A.
   If every two distinct sets in S are disjoint, then the collection S of subsets of a set A is called pairwise disjoint.

3. Let A be a nonempty set. A **partition** of a set A is defined as a collection S of subsets of A satisfying the three properties

   (1) X is not an empty set for every set $X \in S$.

   (2) For every two sets $X, Y \in S$, either $X = Y$ or $X \cap Y = \{\}$. In other words, S is a collection of pairwise disjoint subsets of A.

   (3) $\displaystyle\bigcup_{X \in S} X = A$

Example Let $A = \{x \in \mathbb{N} | 1 \leq x \leq 10\}$.
Let $A_1 = \{1, 4, 10\}$, $A_2 = \{5, 7, 8\}$, $A_3 = \{2, 3, 6, 9\}$ and let $S = \{A_1, A_2, A_3\}$.

A collection $S = \{A_1, A_2, A_3\}$, **whose elements are subsets of** A, satisfies the above three properties in the definition of "*the partitions of sets*":

(1) For every element in S, that is, A_1, A_2, and A_3 are not empty sets.

(2) For every two sets in S,

   Pariwise disjoint : $A_1 \bigcap A_2 = \emptyset$, $A_2 \bigcap A_3 = \emptyset$, $A_3 \bigcap A_1 = \emptyset$.

   That is, every two sets in S are disjoint.

(3) $\bigcup_{i=1}^{3} A_i = A_1 \bigcup A_2 \bigcup A_3 = \{1, 4, 10, 5, 7, 8, 2, 3, 6, 9\}$.

   Since the order in a set doesn't matter, $\{1, 4, 10, 5, 7, 8, 2, 3, 6, 9\} = A$.

Therefore, S is a partition of A.

Here are self–practice problems. The following questions are a continuation of the previous example.

1. Draw a proper Venn Diagram of a partition of the set A.

2. Which of following are true statements?

    a. $\emptyset \in S$

    b. $\emptyset \subset S$

    c. $A_1 \subset S$

    d. $A_3 \in S$

    e. $A_2 \in A$

    f. $A_1 \subset A$

    g. $S \subset \mathcal{P}(A)$

    h. $A \subseteq S$.

    i. $\mathcal{P}(A) - \{\emptyset\}$ is a partition of the set A.