Examples

A. Given the set \( S = \{\{1, 2\}, 3, 4\} \).

(a) List the elements of \( S \).
\[
\{1, 2\},
3,
4
\]

(b) Which of the following are true statements?

(i) \( 2 \in S \)
No, it is not true because \( 2 \) is not an element of the set \( S \). So \( 2 \not\in S \).

(ii) \( \{1, 2\} \in S \)
Yes, it is true.

(iii) \( \{1, 2\} \subseteq S \)
No, here \( \{1, 2\} \) is an element of the set \( S \).
We can write \( \{\{1, 2\}\} \subset S \) and the set \( \{\{1, 2\}\} \) has one element.

(iv) \( \{3, 4\} \subseteq S \)
Yes, it is true.

B. Find the corresponding power sets of the set \( M = \{0, 1\} \) and of the set \( K = \{a, b, c\} \).

\[
\mathcal{P}(M) = \{\emptyset, \{0\}, \{1\}, \{0, 1\}\}.
\]

\[
\mathcal{P}(K) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}.
\]

A power set is again a set! If you answer it as follows

\[
\mathcal{P}(M) = \emptyset, \{0\}, \{1\}, \{0, 1\}
\]

it is not correct because the left hand side is a set and the right hand side is not a set.

C. What is the cardinality of \( \mathcal{P}(M) \)? How about \( |\mathcal{P}(K)| \)?

The cardinality of \( \mathcal{P}(M) \) is 4 and you can also write \( |\mathcal{P}(M)| = 4 \)

\( |\mathcal{P}(K)| = 8. \)

D. Can you make a conjecture how \( A \) and \( |\mathcal{P}(A)| \) are related if \( A \) is a finite set?
Note that the set $M$ has two elements and the cardinality of its power set is 4. Also, the set $P$ has three elements and $|\mathcal{P}(K)| = 8$. We can think that $4 = 2^2$ and $8 = 2^3$. The conjecture would be if a set $A$ has $n$ elements then $|\mathcal{P}(A)| = 2^n$. Why? When we make a subset of a set A, we have two choices for each element in a set A. Either include an element or exclude an element. So, we have $2 \times 2 \times \cdots \times 2 = 2^n$. 