
Examples

A. Given the set $S = \{\{1, 2\}, 3, 4\}$.

(a) List the elements of S.

{1, 2 },
3,
4

(b) Which of the following are true statements?

(i) $2 \in S$
No, it is not true because 2 is not an element of the set S. So $2 \notin S$.

(ii) $\{1, 2\} \in S$
Yes, it is true.

(iii) $\{1, 2\} \subseteq S$
No, here $\{1, 2\}$ is an element of the set S.
We can write $\{\{1, 2\}\} \subset S$ and the set $\{\{1, 2\}\}$ has one element.

(iv) $\{3, 4\} \subseteq S$
Yes, it is true.

B. Find the corresponding power sets of the set $M = \{0, 1\}$ and of the set $K = \{a, b, c\}$.

$\mathcal{P}(M) = \{\emptyset, \{0\}, \{1\}, \{0, 1\}\}$.
$\mathcal{P}(K) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$.

A power set is again **a SET**! If you answer it as follows

$$\mathcal{P}(M) = \emptyset, \{0\}, \{1\}, \{0, 1\}$$

it is not correct because the left hand side is a set and the right hand side is not a set.

C. What is the cardinality of $\mathcal{P}(M)$? How about $|\mathcal{P}(K)|$?

The cardinality of $\mathcal{P}(M)$ is 4 and you can also write $|\mathcal{P}(M)| = 4$
$|\mathcal{P}(K)| = 8$.

D. Can you make a conjecture how A and $|\mathcal{P}(A)|$ are related if A is a finite set?

Note that the set M has two elements and the cardinality of its power set is 4. Also, the set P has three elements and $|\mathcal{P}(K)| = 8$. We can think that $4 = 2^2$ and $8 = 2^3$. The conjecture would be if a set A has n elements then $|\mathcal{P}(A)| = 2^n$. Why? When we make a subset of a set A, we have two choices for each element in a set A. Either include an element or exclude an element. So, we have $2 \times 2 \times \cdots \times 2 = 2^n$.