
Section 9.5

9.42 Prove or disprove the following.

(a) If two functions $f : A \to B$ and $g : B \to C$ are both bijective, then $g \circ f : A \to C$ is bijective.

(c) Let $f : A \to B$ and $g : B \to C$ be two functions. If g is one-to-one, then $g \circ f : A \to C$ is one-to-one.

9.44 Let A denote the set of integers that are multiples of 4, let B denote the set of integers that are multiples of 8 and let B' denote the set of even integers. Thus $A = \{4k : k \in \mathbb{Z}\}$, $B = \{8k : k \in \mathbb{Z}\}$ and $B' = \{2k : k \in \mathbb{Z}\}$. Let $f : A \times A \to B$ and $g : B' \to \mathbb{Z}$ be functions defined by $f((x, y)) = xy$ for $x, y \in A$ and $g(n) = n/2$ for $b \in B'$.

(a) Show that the composition function $g \circ f : A \times A \to \mathbb{Z}$ is defined.

(b) Let For $k, l \in \mathbb{Z}$, determine $(g \circ f)((4k, 4l))$.

9.46 Let A be the set of odd integers and B the set of even integers. A function $f : A \times B \to A \times A$ is defined by $f(a, b) = (3a - b, a + b)$ and a function $g : A \times A \to B \times A$ is defined by $g(c, d) = (c - d, 2c + d)$.

(a) Determine $(g \circ f)(3, 8)$

(b) Determine whether the function $g \circ f : A \times B \to B \times A$ is one-to-one.

(b) Determine whether $g \circ f$ is onto.

Problem 4 Complete the example about cities, states and capitals on page 2 of the lecture notes.