Section 8.1

8.4 Let \( A = \{a, b, c\} \) and \( B = \{1, 2, 3, 4\} \). Then \( R_1 = \{(a, 2), (a, 3), (b, 1), (b, 3), (c, 4)\} \) is a relation from \( A \) to \( B \) while \( R_2 = \{(1, b), (1, c), (2, a), (2, b), (3, c), (4, a), (4, c)\} \) is a relation from \( B \) to \( A \). A relation \( R \) is defined on \( A \) by \( x R y \) if there exists \( z \in B \) such that \( x R_1 z \) and \( z R_2 y \). Express \( R \) by listing its elements.

8.6 A relation \( R \) is defined on \( \mathbb{N} \) by \( a R b \) if \( a/b \in \mathbb{N} \). For \( c, d \in \mathbb{N} \), under what conditions is \( c R^{-1} d \)?

8.10 Let \( A \) be a set with \(|A| = 4\). What is the maximum number of elements that a relation \( R \) on \( A \) can contain so that \( R \cap R^{-1} = \emptyset \)?

Section 8.2

8.12 Let \( S = \{a, b, c\} \). Then \( R = \{(a, a), (a, b), (a, c)\} \) is a relation on \( S \). Which of the properties reflexive, symmetric and transitive does the relation \( R \) possess? Justify your answer.

8.14 Let \( A = \{a, b, c, d\} \). Give an example (with justification) of a relation \( R \) on \( A \) that has no reflexive properties.

8.16 Let \( A = \{a, b, c, d\} \). How many relations defined on \( A \) are reflexive, symmetric and transitive and contain the ordered pairs \((a, b), (b, c), (c, d)\)?

8.22 Let \( S \) be the set of all polynomials of degree at most 3. An element \( s(x) \) of \( S \) can be expressed as \( s(x) = ax^3 + bx^2 + cx + d \), where \( a, b, c, d \in \mathbb{R} \). A relation \( R \) is defined on \( S \) by \( p(x) R q(x) \) if \( p(x) \) and \( q(x) \) have a real root in common. (For example \( p(x) = (x - 1)^2 \) and \( q(x) = x^2 - 1 \) have the root 1 in common so that \( p(x) R q(x) \).) Determine which of the properties reflexive, symmetric and transitive are possessed by \( R \).

Section 8.3

8.24 Let \( R \) be an equivalence relation on \( A = \{a, b, c, d, e, f, g\} \) such that \( a R c, c R d, d R g \) and \( b R f \). If there are three distinct equivalence classes resulting from \( R \), then determine these equivalence classes and determine all elements of \( R \).