Section 4.1

4.2 (4.2 in second edition) Let $a, b \in \mathbb{Z}$, where $a \neq 0$ and $b \neq 0$. Prove that if $a \mid b$ and $b \mid a$, then $a = b$ or $a = -b$.

4.4 (4.4 in second edition) Let $x, y \in \mathbb{Z}$. Prove that if $3 \nmid x$ and $3 \nmid y$, then $3 \mid (x^2 - y^2)$.

4.6 (4.6 in second edition) Let $a \in \mathbb{Z}$. Prove that if $3 \mid 2a$, then $3 \mid a$.

Section 4.2

4.14 (4.10 in second edition) Let $a, b, n \in \mathbb{Z}$, where $n \geq 2$. Prove that if $a \equiv b \pmod{n}$, then $a^2 \equiv b^2 \pmod{n}$.

4.18 (not in second edition) Let $m, n \in \mathbb{N}$ such that $m \geq 2$ and $m \mid n$. Prove that if a and b are integers such that $a \equiv b \pmod{n}$, then $a \equiv b \pmod{m}$.

4.22 (4.16 in second edition) Let $n \in \mathbb{Z}$. Prove each of the following statements.

(a) If $n \equiv 0 \pmod{7}$, then $n^2 \equiv 0 \pmod{7}$.

(b) If $n \equiv 1 \pmod{7}$, then $n^2 \equiv 1 \pmod{7}$.

(c) If $n \equiv 2 \pmod{7}$, then $n^2 \equiv 4 \pmod{7}$.

(d) If $n \equiv 3 \pmod{7}$, then $n^2 \equiv 2 \pmod{7}$.

(e) For each integer n, $n^2 \equiv (7 - n)^2 \pmod{7}$.

(f) For every integer n, n^2 is congruent to exactly one of $0, 1, 2$, or 4 modulo 7.

Section 4.3

4.78 (4.66 in second edition) Prove for every two positive real numbers a and b that $\frac{a}{b} + \frac{b}{a} \geq 2$.

4.89 Prove that for any three integers a, b, c the sum $|a - b| + |a - c| + |b - c|$ is an even integer.

4.90 (not in second edition) Prove that for any numbers $a, b, c, d \in \mathbb{R}$, $ac + bd \leq \sqrt{a^2 + b^2}\sqrt{c^2 + d^2}$

Section 4.4

4.43 (4.30 in second edition) *Note: Since this is an odd numbered problem, give answers to (a) and (b) which are different from the answers in the back of the book. For part (c), the answer in the back is not a complete proof; you are to write a complete proof.)*

(a) Give an example of three sets A, B, and C such that $A \cap B = A \cap C$ but $B \neq C$.

(b) Give an example of three sets A, B, and C such that $A \cup B = A \cup C$ but $B \neq C$.

(c) Let A, B, and C be sets. Prove that if $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then $B = C$.

4.48 (not in second edition) Let $A = \{n \in \mathbb{Z} : 2 \mid n\}$ and $B = \{n \in \mathbb{Z} : 42 \mid n\}$. Prove that $n \in A - B$ if and only if $n = 2k$ for some odd integer k.

4.50 (not in second edition) Prove for every two sets A and B that $A - B$, $B - A$, and $A \cap B$ are pairwise disjoint.

Section 4.5

4.52 (4.34 in second edition) Prove that $A \cap B = B \cap A$ for every two sets A and B (Theorem 4.22(1b)).

4.56 (4.38 in second edition) Let A, B, and C be sets. Prove that $(A - B) \cup (A - C) = A - (B \cap C)$.

Section 4.6

4.62 (4.40 in second edition) Let A and B be sets. Prove that $A \times B = \emptyset$ if and only if $A = \emptyset$ and $B = \emptyset$.

4.64 (4.42 in second edition) For sets A and B, find a necessary and sufficient condition for $(A \times B) \cap (B \times A) = \emptyset$. Verify this condition is necessary and sufficient.

4.68 (4.46 in second edition) Let A, B, C, and D be sets. Prove that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.

4.70 (4.48 in second edition) Let A and B be sets. Show, in general, that $\overline{A \times B} \neq \overline{A} \times \overline{B}$.