Section 2.1
2.A (not in the text) Which of the following are statements? Explain.

1. Let x be a positive integer. Then $\sqrt{x}$ is rational.
2. Mathematics is fun.
3. The President of the United States in 2089 will be a woman.
4. The integer 105 is prime.

2.2 Consider the sets A, B, C, and D below. Which of the following statements are true? Give an explanation for each false statement.

$$A = \{1, 4, 7, 10, 13, 16, \dots\}, \quad C = \{x \in \mathbb{Z} \mid x \text{ is prime and } x \neq 2\},$$

$$B = \{x \in \mathbb{Z} \mid x \text{ is odd}\}, \quad D = \{1, 2, 3, 5, 8, 13, 21, 34, 55, \dots\}$$

(a) $25 \in A$

(b) $33 \in D$

(c) $22 \notin A \cup D$

(d) $C \subseteq B$

(e) $\emptyset \in B \cap D$

(f) $53 \notin C$

2.4 Consider the open sentence $P(x) : x(x-1) = 6$ over the domain $\mathbb{R}$.

(a) For what values of x is $P(x)$ a true statement?

(b) For what values of x is $P(x)$ a false statement?

Section 2.2
2.14 State the negation of each of the following statements.

(a) At least two of my library books are overdue.

(b) One of my two friends misplaced his homework assignment.

(c) No one expected that to happen.

(d) It's not often that my instructor teaches that course.

(e) It's surprising that two students received the same exam score.

Section 2.3

2.16 (2.11 in 2nd edition) For the sets $A = \{1, 2, \ldots, 10\}$ and $B = \{2, 4, 6, 9, 12, 25\}$ consider the following two statements:

$$P : A \subseteq B, \qquad Q : |A - B| = 6.$$

Determine which of the following statements are true.

(a) $P \vee Q$

(b) $P \vee (\sim Q)$

(c) $P \wedge Q$

(d) $(\sim P) \wedge Q$

(e) $(\sim P) \vee (\sim Q)$

2.18 (2.13 in 2nd edition) Let $S = \{1, 2, \ldots, 6\}$ and let

$$P(A) : A \cap \{2, 4, 6\} = \emptyset, \qquad Q(A) : A \neq \emptyset$$

be open sentences over the domain $\mathcal{P}(S)$.

(a) Determine all $A \in \mathcal{P}(S)$ for which $P(A) \wedge Q(A)$ is true.

(b) Determine all $A \in \mathcal{P}(S)$ for which $P(A) \vee (\sim Q(A))$ is true.

(c) Determine all $A \in \mathcal{P}(S)$ for which $(\sim P(A)) \wedge (\sim Q(A))$ is true.

Section 2.4

2.20 (2.15 in 2nd edition) For statement P and Q, construct a truth table for $(P \implies Q) \implies (\sim P)$.

2.24 Two sets A and B are nonempty disjoint subsets of a set S. If $x \in S$, then which of the following are true?

(a) It's possible that $x \in A \cap B$.

(b) If x is an element of A, then x can't be an element of B.

(c) If x is not an element of A, then x must be an element of B.

(d) It's possible that $x \notin A$ and $x \notin B$.

(e) For each nonempty set C, either $x \in A \cap C$ or $x \in B \cap C$.

(f) For some nonempty set C, both $x \in A \cup C$ and $x \in B \cup C$.

2.28 Consider the statement (implication):

If Bill takes Sam to the concert, then Sam will take Bill to dinner.

Which of the following implies that this statement is true?

(a) Sam takes Bill to dinner only if Bill takes Sam to the concert.

(b) Either Bill doesn't take Sam to the concert or Sam takes Bill to dinner.

(c) Bill takes Sam to the concert.

(d) Bill takes Sam to the concert and Sam takes Bill to dinner.

(e) Bill takes Sam to the concert and Sam doesn't take Bill to dinner.

(f) The concert is canceled.

(g) Sam doesn't attend the concert.

Section 2.5

2.32 (2.20 in 2nd edition) In each of the following, two open sentences $P(x)$ and $Q(x)$ over a domain S are given. Determine all $x \in S$ for which $P(x) \implies Q(x)$ is a true statement.

(a) $P(x) : x - 3 = 4$; $Q(x) : x \geq 8$; $S = \mathbb{R}$.

(b) $P(x) : x^2 \geq 1$; $Q(x) : x \geq 1$; $S = \mathbb{R}$.

(c) $P(x) : x^2 \geq 1$; $Q(x) : x \geq 1$; $S = \mathbb{N}$.

(d) $P(x) : x \in [-1, 2]$; $Q(x) : x \leq 2$; $S = [-1, 1]$.

2.34 (a,b,e,f) Each of the following describes an implication. Write the implication in the form "if . . . , then . . . ".

(a) Any point on the straight line with equation $2y + x - 3 = 0$ whose x-coordinate is an integer also has an integer for its y-coordinate.

(b) The square of every odd integer is odd.

(e) Let C be a circle of circumference 4π. Then the area of C is also 4π.

(f) Let $n \in \mathbb{Z}$. The integer n^3 is even only if n is even.

Section 2.6

2.36 (2.25 in 2nd edition) Let $P(x)$: x *is odd.* and $Q(x)$: x^2 *is odd.* be open sentences over the domain $\mathbb{Z}$. State $P(x) \iff Q(z)$ in two ways: (1) using "if and only if" and (2) using "necessary and sufficient."

2.40 (2.27 in 2nd edition) In each of the following, two open sentences $P(x, y)$ and $Q(x, y)$ are given, where the domain of both x and y is $\mathbb{Z}$. Determine the truth value of $P(x) \iff Q(x)$ for the given values of x and y.

(a) $P(x, y) : x^2 - y^2 = 0$ and $Q(x, y) : x = y$. $(x, y) \in \{(1, -1), (3, 4), (5, 5)\}$.

(b) $P(x, y) : |x| = |y|$ and $Q(x, y) : x = y$. $(x, y) \in \{(1, 2), (2, -2), (6, 6)\}$.

(c) $P(x,y) : x^2+y^2 = 1$ and $Q(x,y) : x+y = 1$. $(x,y) \in \{(1,-1),(-3,4),(0,-1),(1,0)\}$.

2.44 (2.29 in 2nd edition) Let $S = \{1,2,3,4\}$. Consider the following open sentences over the domain S:

$$P(n) : \frac{n(n-1)}{2} \text{ is even.}$$

$$Q(n) : 2^{n-2} - (-2)^{n-2} \text{ is even.}$$

$$R(n) : 5^{n+1} + 2^n \text{ is prime.}$$

Determine four distinct elements a,b,c,d in S such that all of the following are satisfied.

(i) $P(a) \Rightarrow Q(a)$ is false;

(ii) $Q(b) \Rightarrow P(b)$ is true;

(iii) $P(c) \iff R(c)$ is true;

(iv) $Q(d) \iff R(d)$ is false.

Section 2.7

2.46 (2.30 in 2nd edition) For statements P and Q, show that $P \Rightarrow (P \vee Q)$ is a tautology.

Section 2.8

2.52 (2.35 in 2nd edition) Let P and Q be statements.

(a) Is $\sim (P \vee Q)$ logically equivalent to $(\sim P) \vee (\sim Q)$? Explain.

(b) What can you say about the biconditional $\sim (P \vee Q) \iff ((\sim P) \vee (\sim Q))$?

2.54 (2.37 in 2nd edition) For statements P and Q, show that $(\sim Q) \implies (P \wedge (\sim P))$ and Q are logically equivalent.

Section 2.9

2.58 (2.39 in 2nd edition) Verify the following laws stated in Theorem 2.18:

(a) Let P, Q, and R be statements. Then

$$P \vee (Q \wedge R) \text{ and } (P \vee Q) \wedge (P \vee R) \text{ are logically equivalent.}$$

(b) Let P and Q be statements. Then

$$\sim (P \vee Q) \text{ and } (\sim P) \wedge (\sim Q) \text{ are logically equivalent.}$$

2.60 (2.41 in 2nd edition) Consider the implication: If x and y are even, then xy is even.

(a) State the implication using "if and only if".

(b) State the converse of the implication.

(c) State the implication as a disjunction (see Theorem 2.17).

(d) State the negation of the implication as a conjunction (see Theorem 2.21(a)).

2.62 Let P and Q be statements. Show that $[(P \vee Q) \wedge \sim (P \wedge Q)] \equiv \sim (P \iff Q)$.

Section 2.10

2.68 (2.46 in 2nd edition; additional instructions given in boldface) State the negations of the following quantified statements **and re-state the original statement using symbols**:

(a) For every rational number r, the number $1/r$ is rational.

(b) There exists a rational number r such that $r^2 = 2$.

2.70 (c,e,f) (2.48 in 2nd edition) Determine the true value of each of the following statements.

(c) $\forall x \in \mathbb{R}, \sqrt{x^2} = x$

(e) $\exists x \in \mathbb{R}, \exists y \in \mathbb{R}, x + y + 3 = 8$

(f) $\forall x, y \in \mathbb{R}, x + y + 3 = 8$

2.74 (2.50 in 2nd edition) Consider the open sentence

$$P(x, y, z) : (x - 1)^2 + (y - 2)^2 + (z - 2)^2 > 0$$

where the domain of each of the variables x, y, and z is $\mathbb{R}$.

(a) Express the quantified statement $\forall x \in \mathbb{R}$, $\forall y \in \mathbb{R}$, $\forall z \in \mathbb{R}$, $P(x, y, z)$ in words.

(b) Is the quantified statement in (a) true or false? Explain.

(c) Express the negation of the quantified statement in (a) in symbols.

(d) Express the negation of the quantified statement in (b) in words.

(e) Is the negation of the quantified statement in (a) true or false? Explain.

2.78 Consider the open sentence $P(a, b) : a/b < 1$ where the domain of a is $A = \{3, 5, 8\}$ and the domain of b is $B = \{3, 6, 10\}$.

(a) State the quantified statement $\forall a \in A, \exists b \in B, P(a, b)$ in words.

(b) Show the quantified statement in (a) is true.