Replace the definition on page 203 with the definition presented here.

**Definition 1.** Let \( f \) be defined on an interval \( I \).
1. Then \( f \) is concave up on \( I \) means for each pair \( a < b \in I \), the graph of \( f \) between \( a \) and \( b \) lies below the line segment joining \((a, f(a))\) and \((b, f(b))\).
2. Then \( f \) is concave down on \( I \) means for each pair \( a < b \in I \), the graph of \( f \) between \( a \) and \( b \) lies above the line segment joining \((a, f(a))\) and \((b, f(b))\).

It should be obvious that \( f \) is concave up on an interval \( I \) if and only if \(-f\) is concave down on \( I \). What isn’t so obvious is that a function can be concave up on an interval without being differentiable everywhere on that interval. For example the function defined by

\[
f(x) = \begin{cases} 
  x^2 & \text{if } x \leq 0 \\
  (x + 1)^3 - 1 & \text{if } x > 0
\end{cases}
\]

is concave up on \((-\infty, \infty)\) which can easily be seen by sketching its graph. However, \( \lim_{x \to 0^-} \frac{f(x)-0}{x-0} = \frac{d}{dx} x^2 \bigg|_{x=0} = 0 \) while \( \lim_{x \to 0^+} \frac{f(x)-0}{x-0} = \frac{d}{dx} ((x + 1)^3 - 1) \bigg|_{x=0} = 1 \). Consequently \( f'(0) \) doesn’t exist. But if \( f \) is differentiable, then it’s possible to determine whether or not the function is concave up or down from the derivative as the following theorem indicates.

**Theorem 1.** Let \( f \) be continuous on an interval \( I \) and differentiable on the interior of \( I \).
1. If \( f' \) is increasing on the interior of \( I \), then \( f \) is concave up on \( I \).
2. If \( f' \) is decreasing on the interior of \( I \), then \( f \) is concave down on \( I \).

The next test for concavity is a consequence of Corollary 3 on page 119.

**Corollary 1.** Let \( f \) be continuous on an interval \( I \) and twice differentiable on the interior of \( I \).
1. If \( f''(x) > 0 \) for each \( x \) in the interior of \( I \), then \( f \) is concave up on \( I \).
2. If \( f''(x) < 0 \) for each \( x \) in the interior of \( I \), then \( f \) is concave down on \( I \).