Supplement 5 for Section 3.6

This material should come at the end of page 146.

The Power Rule will now be extended to rational exponents using the Chain Rule. First recall that a rational number is one that can be written as a quotient of two integers. For example $\frac{8}{6}$. The representation isn’t unique. In this case $\frac{8}{6} = \frac{-4}{3}$. The representation $r = \frac{m}{n}$ is unique if $n$ is required to be positive and if $m$ and $n$ have no common divisors except for 1. In that case if $n$ is even, the domain of the function $f(x) = x^{\frac{m}{n}}$ is $[0, \infty)$ if $m \geq 0$ or $(0, \infty)$ otherwise. But if $n$ is odd, then the domain of $f$ is $(-\infty, \infty)$ if $m \geq 0$ or $(-\infty, 0) \cup (0, \infty)$ otherwise.

**More General Power Rule.** Let $r$ be a rational number. Then for any $x$ in the domain of $x^r$

$$\frac{d}{dx} x^r = rx^{r-1}$$

**Proof.** Choose a positive integer, $n$, and an integer, $m$, so that $r = \frac{m}{n}$. Then

$$\frac{d}{dx} x^r = \frac{d}{dx} x^\frac{m}{n} = \frac{d}{dx} \left( x^{\frac{1}{n}} \right)^m$$

$$= m \left( x^{\frac{1}{n}} \right)^{m-1} \frac{d}{dx} x^{\frac{1}{n}} \quad \text{(by the Chain Rule)}$$

$$= m \left( x^{\frac{m-1}{n}} \right) \frac{1}{n} x^{\frac{1}{n}-1}$$

$$= \frac{m}{n} x^{\frac{m}{n}-\frac{1}{n}+\frac{1}{n}-1} = rx^{r-1}$$