2.1B Solving First-Degree Equations

A. Equation Rule

You can do anything (add/subtract/multiply/divide) to one side of an equation, so long as you do it to the other side.

For instance, suppose we have the equation $3x^2 + 71 = 4x^2 + 10$.

We could, if we wanted, subtract 4 from both sides: $3x^2 + 71 \underline{-4} = 4x^2 + 10 \underline{-4}$.

Note: In this situation, subtracting 4 doesn't help much. Our goal is to make good choices when we solve equations.

We will now look at solution techniques for equations with one variable having no powers. These are called **first-degree equations**.

B. Method for Solving First-Degree Equations

1. Simplify each side:

    a. clear parentheses (distributive property!)

    b. clear fractions (multiply both sides by the LCD)

    c. collect like terms

2. Isolate the variable:

    This means: add/subtract variables to get the variables on one side, and add/subtract numbers to get the numbers on the other side

3. Get x by itself

4. Check your answer.

C. Examples

Example 1: Solve $3 - 5x - 2 = 2x + 15$ for x

Solution

1. No parentheses, no fractions; collect like terms: $1 - 5x = 2x + 15$

2. Isolate x: "move $2x$ to the left and 1 to the right"

$$1 - 5x \underline{-2x} = 2x + 15 \underline{-2x}$$

$$1 - 7x = 15$$

$$1 - 7x \underline{-1} = 15 \underline{-1}$$

$$-7x = 14$$

3. Get x by itself: divide both sides by -7

$$\frac{-7x}{-7} = \frac{14}{-7}$$

$$x = \frac{14}{-7} = -2$$

4. Check it: plug it back into the **original** equation

$$3 - 5(-2) - 2 \stackrel{?}{=} 2(-2) + 15$$

$$3 + 10 - 2 \stackrel{?}{=} -4 + 15$$

$$13 - 2 \stackrel{?}{=} 11$$

It checks!

Ans $\boxed{x = -2}$

Example 2: Solve $2x - 5(x - 2) = 6 - 8x + 1$ for x

Solution

1. Simplify:

   Clear parentheses: $2x - 5x + 10 = 6 - 8x + 1$

   No fractions to clear

   Collect like terms: $-3x + 10 = 7 - 8x$

2. Isolate x: "move $-8x$ to the left and 10 to the right"

$$-3x + 10 \underline{+8x} = 7 - 8x \underline{+8x}$$

$$5x + 10 = 7$$

$$5x + 10 \underline{-10} = 7 \underline{-10}$$

$$5x = -3$$

3. Get x by itself: divide both sides by 5

$\frac{5x}{5} = \frac{-3}{5}$

$x = -\frac{3}{5}$

4. Check it: plug it back into the **original** equation

$2(-\frac{3}{5}) - 5(-\frac{3}{5} - 2) \stackrel{?}{=} 6 - 8(-\frac{3}{5}) + 1$

$-\frac{6}{5} - 5(-\frac{3}{5} - \frac{10}{5}) \stackrel{?}{=} 6 + \frac{24}{5} + 1$

$-\frac{6}{5} - 5(-\frac{13}{5}) \stackrel{?}{=} \frac{30}{5} + \frac{24}{5} + \frac{5}{5}$

$-\frac{6}{5} + \frac{65}{5} \stackrel{?}{=} \frac{59}{5}$

It checks!

Ans $\boxed{x = -\frac{3}{5}}$

Example 3: Solve $\frac{3x}{4} - \frac{1}{6} = \frac{2x}{3} + 2$ for x

Solution

1. Simplify:

No parentheses to clear

Clear fractions: multiply both sides by LCD$= 12$

$$12\left(\tfrac{3x}{4} - \tfrac{1}{6}\right) = 12\left(\tfrac{2x}{3} + 2\right)$$

$$9x - 2 = 8x + 24$$

No like terms to combine

2. Isolate x: "move $8x$ to the left and -2 to the right"

$$9x - 2 \underline{-8x} = 8x + 24 \underline{-8x}$$

$$x - 2 = 24$$

$$x - 2 \underline{+2} = 24 \underline{+2}$$

$$x = 26$$

3. x is already by itself: $x = 26$

4. Check it: plug it back into the **original** equation

$$\tfrac{3(26)}{4} - \tfrac{1}{6} \stackrel{?}{=} \tfrac{2(26)}{3} + 2$$

$$\tfrac{78}{4} - \tfrac{1}{6} \stackrel{?}{=} \tfrac{52}{3} + 2$$

$$\tfrac{234}{12} - \tfrac{2}{12} \stackrel{?}{=} \tfrac{208}{12} + \tfrac{24}{12}$$

$$\tfrac{232}{12} \stackrel{?}{=} \tfrac{232}{12}$$

It checks!

Ans $\boxed{x = 26}$

Example 4: Solve $\frac{5x}{3} - \frac{4x}{10} + 3$ for x

Solution

This is **not** an equation! Multiplying by 30 is **not** valid. Don't even think about it!

Move on to the next problem.

Example 5: Solve $\frac{2}{3}(2x + 12) = \frac{5}{6}x$ for x

Solution

1. Simplify:

   Clear parentheses: $\frac{4}{3}x + 8 = \frac{5}{6}x$

   Clear fractions: multiply both sides by LCD$= 6$

   $$6\left(\frac{4}{3}x + 8\right) = 6\left(\frac{5}{6}x\right)$$

   $$8x + 48 = 5x$$

   No like terms to combine

2. Isolate x: "move $5x$ to the left and 48 to the right"

   $$8x + 48 \underline{-5x} = 5x \underline{-5x}$$

   $$3x + 48 = 0$$

   $$3x + 48 \underline{-48} = \underline{-48}$$

   $$3x = -48$$

3. Get x by itself: divide by 3

$$\frac{3x}{3} = \frac{-48}{3}$$

$$x = -16$$

4. Check it: plug it back into the **original** equation

$$\frac{2}{3}\left(2(-16) + 12\right) \stackrel{?}{=} \frac{5}{6}(-16)$$

$$\frac{2}{3}(-32 + 12) \stackrel{?}{=} \frac{\cancel{5}}{\cancel{6}_{3}} \cdot \frac{\cancel{-16}^{-8}}{1}$$

$$\frac{2}{3}(-20) \stackrel{?}{=} -\frac{40}{3}$$

It checks!

Ans $\boxed{x = -16}$