Section 6: More on Graphing

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Parametric Equations

Maple’s plot command can also be used to graph curves described by parametric equations.

To graph the parametric curve corresponding to the pair of parametric equations: \( x = f(t) \) and \( y = g(t) \) on the parameter interval \([a, b]\) use the command:

\[
\text{plot([f(t), g(t), t = a..b], x = xmin..xmax, y = ymin..ymax]);}
\]

There are two things to take careful note of here. First note that there are three entries in the square brackets: the two parametric expressions for \( x \) and \( y \) and the parameter domain. Also note that the viewing window for the plot is separately specified by the \( x \)- and \( y \)-ranges (i.e., \( x = xmin \). \( xmax, y = ymin..ymax)\).

> 

Example 1

Plot the parametric curve determined by \( x = t^2 - t \) and \( y = 2t - t^3 \) over the \( t \)-interval \([-2, 2]\).

> plot([t^2-t,2*t-t^3,t=-2..2],x=-2..5,y=-5..5);

Exercise 6.1

Plot the parametric curve defined \( x = \sin(3t) \) and \( y = \sin(4t) \) over the \( t \)-interval \([0, 2 \pi]\).

For a viewing window let \( x \) and \( y \) range between -2 and 2.

Student Workspace 6.1

> >

Answer 6.1

> plot([sin(3*t),sin(4*t),t=0..2*Pi],x=-2..2,y=-2..2);

Implicit Plots

Maple can plot curves that are implicitly defined by an equation in the variables \( x \) and \( y \).

Example 1

To plot the graph of the hyperbola given by the equation: \( \frac{x^2}{4} - \frac{y^2}{9} = 1 \) use the implicitplot command. To use this command we must first load the "plots" package using the "with" command.
\begin{verbatim}
> with(plots):
> implicitplot(x^2/9 + y^2/4 = 1, x=-5..5, y=-5..5);

\textbf{Example 2}

Graph the equation $\frac{x^2}{25} + \frac{y^2}{9} = 1$ using the \texttt{implicitplot} command.

Recall that this is the equation of an ellipse with the lengths of major and minor axes equal to 10 and 6 respectively.

Our first attempt at getting the expected graph comes up short!

\begin{verbatim}
> implicitplot(x^2/25+y^2/9=1,x=-5..5,y=-5..5);
> Why did we get a circle instead of an ellipse?

The problem here is that the $x$- and $y$-scales are not equal. To force equal scaling add "scaling=constrained" or click on the graph to expose the graphing toolbar, and select the button marked 1:1.

The graph then appears as seen in the following figure.

\begin{verbatim}
> implicitplot(x^2/25+y^2/9=1,x=-5..5,y=-5..5,scaling=constrained);
\end{verbatim}
\end{verbatim}

\textbf{Exercise 6.2}

Graph the equation $x^2 + 4y^2 = 4$

\textbf{Student Workspace 6.2}

\begin{verbatim}
> implicitplot(x^2+4*y^2=4,x=-3..3,y=-2..2,scaling=constrained);
\end{verbatim}

\textbf{Answer 6.2}

\begin{verbatim}
> implicitplot(x^2+4*y^2=4,x=-3..3,y=-2..2,scaling=constrained);
\end{verbatim}

\textbf{Polar Graphs (optional)}

Graphs of polar equations $r = f(\theta)$ are handled by the \texttt{polarplot} command, which is part of the \texttt{plots} package accessed using \texttt{with(plots)}.
Here are some examples. Note that we include the option \texttt{scaling=constrained} to get geometric perspective.

\begin{verbatim}
> polarplot(1+\cos(\theta), \theta=-\Pi..\Pi, scaling=constrained);
> polarplot(\sin(3*\theta), \theta=-\Pi..\Pi, scaling=constrained);
\end{verbatim}

Another way of graph polar graphs is to use the plot option \texttt{coords=polar} and graph the curve using parametric equations. The general form of the command is:

\begin{verbatim}
plot([r(s), \theta(s), s=a..b], coords=polar);
\end{verbatim}

If the parameter \texttt{s} is actually the angle \theta, the command becomes:

\begin{verbatim}
plot([r(\theta), \theta, \theta=a..b], coords=polar);
\end{verbatim}

For example, to graph \(1+\cos(\theta)\) in polar coordinates using the \texttt{plot} command, type:

\begin{verbatim}
> plot([1+\cos(\theta), \theta, \theta=-\Pi..\Pi], coords=polar);
\end{verbatim}

The \texttt{coords=polar} option can be applied to \texttt{implicitplot} command as well.

For example, to graph the lemniscate \(r^2 = 4 \cos(2\theta)\) over the \(\theta\) interval \(-\pi..\pi\), type:

\begin{verbatim}
> implicitplot(r^2=4*\cos(2*\theta), r=0..2, \theta=0..2*\Pi, coords=polar, scaling=constrained, grid=[50,50]);
\end{verbatim}

\textbf{Plot Options}

There are many options available when you use the \texttt{plot} command. To see a list, execute the next line to go directly to Maple's Help Page on this command. Skip this if you wish.

\begin{verbatim}
> ?plot[options];
\end{verbatim}