The first thing to do is go over the previous exams and make sure you know how to do all the problems thereon! You have the solution sheets to guide you. Use these materials as a study outline to guide you through the relevant sections of the book. Then make sure you know how to do the problems assigned for homework. Below is a list of topics we’ve covered, with some stream-of-consciousness thoughts on the most important conceptual relationships between them.

§1.2 (a) First order: explicit and implicit solutions. Be sure you know the difference between verifying that a given function of the independent variable is a solution to a DE, and finding a solution.

(b) Initial Value Problems: What’s the difference between solution to an IVP and general solution of the associated DE? Understand the existence-uniqueness theorem. In particular: This theorem says that “phase plane trajectories never cross.” Why? Does this contradict what you found in #10, page 274? Does it contradict what happens on the “separatrices” \(y = \pm 2|\cos x/2|\) that we found in the phase plane diagram of the nonlinear pendulum? Make sure you understand this point!

§1.3 Direction Fields: make sure you know how to sketch these using the “method of isolines”. Especially be certain you understand how to do this for the autonomous case \(y' = f(y)\).

§1.4 Phase line: compare and contrast with “phase plane.”

§2.2 Separable equations. Good problems to work on are: #29 (uniqueness, or lack thereof) #32 (interval of existence for solutions), #33 (mixing problem) #38 (free fall: solve the DE in three ways—separable, linear via integrating factor, Laplace transform).

§2.3 First order linear equations via integrating factor. Existence and uniqueness of solutions compare with the nonlinear case. How are the theorems different? The same?

§2.4 Exact equations. Good problems to work are: #32, #33 on orthogonal trajectories. The book makes a big deal out of this, but it’s really quite simple: If a curve family is described by the differential equation \(y' = f(x, y)\) then the orthogonal family is described by \(y' = -1/f(x, y)\) (slope of an orthogonal line equals negative reciprocal of slope of original line).\(^1\)

§3.2 Applications. Population models: Malthusian (exponential) and logistic. Know how to analyze logistic equation both via slope fields and via analytic solution.

§4.2–4.9 Second order linear DE’s, including method of variation of parameters. Note that for first order equations the “integrating factor method” is really a simplified version of variation of parameters. Understand the significance of “fundamental sets”.

\(^1\)If you’re into sailing, you might explain to yourself how finding 45° trajectories is relevant to your sport. For extra credit: find the DE for such trajectories given that the original family of curves has DE \(y' = f(x, y)\).
§4.11 Free vibrations. Understand how to interpret the DE \( ay'' + by' + cy = 0 \) for \( a, b, c > 0 \) as a mass-spring system. Know what the coefficients \( a, b, c \) represent. Know the difference between overdamped, underdamped, critically damped. How are these situations reflected in behavior of solutions. Always think of these three DE’s: \( y'' + 2y' + y = 0 \), \( y'' + 3y' + y = 0 \), \( y'' + y' + y = 0 \).

§4.12 Forced vibrations. We didn’t cover this section directly, but did analyze simple cases of the most important topic it discusses: Resonance. Know the difference between behavior of \( y'' + y = \sin x \) and \( y'' + y = \sin 2x \). In Chapter 7 we discussed Resonance from the “unit impulse” point of view (Problem 30, page 435 and handout that amplified this topic). For \( y'' + y = g(x) \), compare the solution you got for \( g(x) = \sin x \) with the one you got for \( g(x) = \sum_{k=0}^{\infty} \delta(x - 2\pi k) \). Think about the analogy between \( g(x) = \sin 2x \) and \( g(x) = \sum_{k=0}^{\infty} \delta(x - \pi k) \) . . .

§5.2 Phase plane analysis of two by two systems. Understand thoroughly the examples we’ve worked: linear pendulum, nonlinear pendulum, predator-prey equations. Know how to prove trajectories are closed. Understand significance of critical points of systems of DE’s and associated equilibrium solutions. Understand significance of critical point on a closed curve containing a trajectory. Not on such a curve. Understand what it means for a critical point to be a center, stable, unstable.

§5.3 Elimination method for systems. Compare with Laplace transform method. Understand the difference between the methods as they relate to Problem 4 of Exam III.

§6.3 Annihilator method for finding particular solutions to nonhomogeneous linear DE’s. Know how to use this for second order equations. It gives rise to higher order DE’s, which is why it’s in Chapter 6.

§7.2–7.8 Laplace transforms, Convolution, transfer and unit impulse functions, Dirac Delta. Understand and know how to use — everything!