1. A tank contains 1180L of pure water solution that contains 0.03kg of sugar per liter enters the tank at the rate 2L/min and is thoroughly mixed into it. The new solution drains out of the tank at the same rate.

(a) How much sugar is in tank at the beginning?
\( y(0) = [ ] \) kg;

(b) Find the amount of sugar after \( t \) minutes
\( y(t) = [ ] \) kg

(c) As \( t \) becomes large, what value is \( y(t) \) approaching? In other words, calculate the following rate
\( \lim_{t \to \infty} = [ ] \) kg.

Solution: Let \( y(t) \) be the amount of salt in the tank at time \( t \), \( y_{in} \) be the amount that has flowed into tank at time \( t \), and \( y_{out} \) be the amount that has flowed out of tank at time \( t \). Since the number of liters flowing into the tank equals the number of liters flowing out of the tank, the total number of liters remains fixed at 1180L. \( y(0) = 0 \) because it is just pure water at the beginning. Hence

\[ y(t) = y_{in}(t) - y_{out}(t), \text{ so } y'(t) = y'_{in}(t) - y'_{out}(t). \]

Notice \( y'_{in}(t) \) is the rate of change of the amount flowing into tank at time \( t \), so

\[ y'_{in}(t) = 2L/min \times 0.03kg/L = 0.06kg/min. \]

Similarly

\[ y_{out} = 2L/min \times y(t)/1180kg/L = y(t)/590kg/min, \]

then

\[ y'(t) = 0.06 - y(t)/590. \]

so \( u(t) = e^{t/590} \) and

\[ y(t) = e^{-t/590}(0.06 \times \int e^{t/590} dt + c) \]
\[ = 35.4 + c \times e^{-t/590} \]

By \( y(0) = 0 \) we can get \( c = -35.4 \). Hence

\[ y(t) = 35.4 - 35.4e^{-t/590}, \]

\( \lim_{t \to \infty} y(t) = 35.4 \)
2. An unknown radioactive element decays into non-radioactive substances. In 200 days the radioactivity of a sample decreases by 49 percent.

(a) What is the half-life of the element?

half-life: [_____] days

(b) How long will it take for a sample of 100mg to decay to 90mg?

time needed: [_____] days.

Solution: Let $Q(t)$ denote the amount at time $t$. $Q_0 = Q(0)$. Then

$$Q(t) = Q_0 e^{-rt},$$

where $r$ is a positive number.

In 200 days the radioactivity decreases by 49%, so $Q(200) = (1 - 49\%)Q_0 = 51\%Q_0$.

i.e

$$Q(200) = 51\%Q_0 = Q_0 e^{-200r},$$

so $r = 0.0033667$. Then for (a),

$$Q_0/2 = Q_0 e^{-0.0024715t},$$

half-life $t = 205.88$.

for (b) $Q_0 = 100, Q(t) = 90$, so

$$90 = 100 e^{-0.0033667t},$$

we get $t = 31.295$. 
3. A tank contains 1720L of pure water solution that contains 0.08kg of sugar per liter enters the tank at the rate 7L/min and is thoroughly mixed into it. The new solution drains out of the tank at the same rate.

(a) How much sugar is in tank at the beginning?
\[ y(0) = \square \text{ kg}; \]

(b) Find the amount of sugar after \( t \) minutes
\[ y(t) = \square \text{ kg} \]

(c) Find the amount of the sugar after 54 minutes.
\[ y(54) = \square \text{ kg}. \]

Solution: Let \( y(t) \) be the amount of salt in the tank at time \( t \), \( y_{\text{in}} \) be the amount that has flowed into tank at time \( t \), and \( y_{\text{out}} \) be the amount that has flowed out of tank at time \( t \). Since the number of liters flowing into the tank equals the number of liters flowing out of the tank, the total number of liters remains fixed at 1720L. \( y(0) = 0 \) because it is just pure water at the beginning.

\[ y(t) = y_{\text{in}}(t) - y_{\text{out}}(t), \text{ so } y'(t) = y'_{\text{in}}(t) - y'_{\text{out}}(t). \]

Notice \( y'_{\text{in}}(t) \) is the rate of change of the amount flowing into tank at time \( t \), so
\[ y'_{\text{in}}(t) = 7 \text{L/min} \times 0.08 \text{kg/L} = 0.56 \text{kg/min}. \]

Similarly
\[ y_{\text{out}} = 7 \text{L/min} \times y(t)/1720 \text{kg/L} = 7y(t)/1720 \text{kg/min}, \]
then
\[ y'(t) = 0.56 - 7y(t)/1720. \]

So \( u(t) = e^{7t/1720} \) and
\[ y(t) = e^{-7t/1720} \left(0.56 \times \int e^{7t/1720} \text{dt} + c \right) = 137.6 + c \times e^{-7t/1720} \]

By \( y(0) = 0 \) we can get \( c = -137.6 \). Hence
\[ y(t) = 137.6 - 137.6e^{-7t/1720}, \]
\[ y(54) = 137.6 - 137.6e^{-7 \times 54/1720} = 27.148 \]