Name: _____ **Section:** _____

MATH 235 – EXAM #1 – Solutions – Newhouse

NO CALCULATORS, NO CELL PHONES, SHOW ALL WORK

Total pts = 60

1. Find the general solution to each of the following differential equations.

   (a) $y' = t^3(y^2 - 1)$

   6 total pts

   2 pts for separation

   $$\frac{dy}{y^2 - 1} = t^3 dt$$

   4 pts for correct integrals plus constant

   $$\frac{1}{2} log(\frac{y-1}{y+1}) = \frac{t^4}{4} + C$$

   (b) $y' + 2t^2 y = 3t^2$
   6 pts total

   2 pts for correct integrating factor $\mu = exp(\frac{2}{3}t^3)$.
   4 pts for correct answer with constant

   $$y = \frac{3}{2} + C\ exp\left(-\frac{2}{3}t^3\right)$$

Name: _____ **Section:** _____

(c) $y' + \frac{y}{t} = ty^3$
total 8 pts

1 pts for recognition as Bernoulli with substitution $v = y^{-2}$

1 pt for correct linear equation for v: $v' - (2/t)v = -2t$

4 pts for solution to linear equation $v = t^2(C - 2 * log(t))$

2 pts for conversion back to y: $y = [t^2(C - 2 * log(t))]^{-\frac{1}{2}}$

(d) $(3x^2y^2 + x)dx + (2x^3y + y^2)dy = 0.$
total 8 pts

2 pts for check $M_y = N_x$ (or if just did correct method, add 2 pts to below)

2 pts for $f(x, y) = x^3y^2 + x^2/2 + g(y)$

2 pts for plugging in $f_y = N$ to get $g'(y) = y^2$
2 pts for ans: $x^3y^2 + x^2/2y^3/3 = C$

Name: _____ **Section:** _____

(e) $(9y^2 + 3 + 4xy^3)dx + (6xy + 3x^2y^2)dy = 0$

10 pts total

4 pts for correct integrating factor: $\mu = x^2$

6 pts for finding correct solution: $x^4y^3 + 3x^3y^2 + x^3 = C$

2. The differential equation $y' = \dfrac{2xy}{x^2 + y^2}$ can be changed to a separable differential equation by an appropriate substitution. Write both this substitution and the resulting separable differential equation. (You do not have to solve the resulting differential equation).

4 pts total

2 pts for correct substitution: $y = xv$

2 pts for correct separable equation: $v + xv' = \frac{2v}{1+v^2}$

Name: _____ **Section:** _____

3. A certain radioactive substance decays from 10g to 5g in 20 years.

   (a) Write a differential equation for the amount of the radioactive substance at time t.

   4 pts total

   2 pts for correct equation: If $Q(t)$ is amount at time t, then $Q' = -kQ$

   2 pts for finding k: $5 = 10exp(-k20)$
   $\frac{1}{2} = exp(-k20),\ -log(2) = -k20,\ k = \frac{log(2)}{20}$

   (b) How long would it take for the substance to decay from 10g to 2g?

   4 pts total

   4 pts for t_1 with $Q(t_1) = 2$,

$$2 = 10exp(-kt_1)$$

$$1/5 = exp(-kt_1)$$

$$-log(5) = -kt_1$$

$$t_1 = \frac{log(5)}{k} = \frac{20\ log(5)}{log(2)}\text{years}$$

Name: _____ **Section:** _____

4. A 300 gallon tank contains 30 gallons of a 5 mg/gal salt solution. A solution with 2 mg/gal is introduced at the rate of 3 gal/min and the resulting solution is drained at the same rate.

   (a) Write a differential equation for the amount $Q(t)$ of salt in the resulting solution at time t.

   4 pts total

   4 pts for DE: Let $Q(t)$ be the amount of salt in the tank at time t.

$$\begin{aligned} Q' &= Q'_{in} - Q'_{out} \\ &= 6 - 3\frac{Q}{30} \\ &= 6 = \frac{Q}{10} \end{aligned}$$

$$Q' + \frac{Q}{10} = 6$$

   (b) How much salt is in the combined mixture after 10 minutes?

   6 pts total

   4 pts for general solution of ODE:

$$Q = e^{-t/10}\left[60e^{t/10} + C\right] = 60 + Ce^{-t/10}$$

   2 pts for determination of C: $Q_0 = 150 = 60 + C,\ C = 90$

   2 pts for $Q(10)$: $Q(10) = (60 + \frac{90}{e})$ mg