WARMUP PROBLEMS (Not to be turned in)

1. Textbook problems:
   
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2. Prove one of the following identities concerning unions and intersections of sets:

   (a) \( A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \)
   
   (b) \( A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \)

3. Given two sets, \( A \) and \( B \), prove one of the following identities:

   (a) \( (A \cup B)^c = A^c \cap B^c \)
   
   (b) \( (A \cap B)^c = A^c \cup B^c \)

   Note: These identities hold for arbitrary unions and intersections.

HOMEWORK EXERCISES

1. Textbook exercises:

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2. Suppose \( E \subseteq \mathbb{R} \) and \( a, b \in E \), with \( a < b \).

(a) Recall that \([a, b] \subseteq E\) if and only if \( \forall x \in [a, b], x \in E \). The negation of this "if and only if" statement is given by:

\([a, b] \nsubseteq E\) if and only if \( \neg(\forall x \in [a, b], x \in E) \).

Expand the right hand side of the new (logically equivalent) statement.

(b) If \( E \) is connected, show that \([a, b] \subseteq E\).

3. Let \( \{U_\alpha\}_{\alpha \in A} \) be a collection of non-empty sets.

(a) Determine the truth of each of the following statements:

i. If \( x \in \bigcup_{\alpha \in A} U_\alpha \), then \( \exists \alpha \in A \) such that \( x \in U_\alpha \).

ii. If \( x \in \bigcup_{\alpha \in A} U_\alpha \), then \( \forall \alpha \in A, x \in U_\alpha \).

iii. If \( x \in \bigcap_{\alpha \in A} U_\alpha \), then \( \exists \alpha \in A \) such that \( x \in U_\alpha \).

iv. If \( x \in \bigcap_{\alpha \in A} U_\alpha \), then \( \forall \alpha \in A, x \in U_\alpha \).

For each case that is false, present a counterexample.

(b) Write the converse of each statement in (a), and repeat.

4. If \( A \) is non-empty subset of \( \mathbb{R}^n \), we define the \textit{distance} from a point \( x \in \mathbb{R}^n \) and \( A \) as

\[
d(x, A) := \inf_{a \in A} \{\|x - a\|\}.
\]

(a) Show that if \( x \in A \), then \( d(x, A) = 0 \). In fact, a stronger result holds:

(b) Show that \( x \in \overline{A} \) if and only if \( d(x, A) = 0 \).