Directions:

- All problems should be collocated, stapled and contain a single cover sheet with your name and a list of problems.
- Clearly mark one problem which you would like to have graded. Additional problem(s) will be selected randomly.
- For each problem, submit only the final version of your solution. There should be enough details describing how you arrived at your solution, but do not submit “scratchwork” containing unnecessary computations.
- Homework is due at the start of class on the due date.

WARMUP PROBLEMS (Not to be turned in)

1. If $f : [a, b] \rightarrow \mathbb{R}$, give two equivalent definitions for what it means for $f$ to be (Riemann) integrable.

2. Give a precise statement for what it means for $f$ to be continuous on an interval $[a, b]$.

3. Give a precise statement for what it means for $f$ to be uniformly continuous on an interval.

4. True/False. If $f(x) > 0$ for all $x \in I$, where $I$ is some interval, then $\inf_{x \in I} f(x) > 0$.

5. Consider the partition $P = \{0, \frac{\pi}{2}, \pi\}$ of $[0, \pi]$. If $f(x) = \sin(x)$, compute the following:
   (a) $U(f, P)$ and $L(f, P)$.
   (b) $S(f, P, t_j)$, where the sampling is given by $\{\frac{\pi}{4}, \frac{3\pi}{4}\}$.

HOMEWORK EXERCISES

1. Textbook exercises:

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2. Consider the function \( f: \mathbb{R} \to \mathbb{R} \) defined by

\[
f(x) = \begin{cases} 
x^2 \sin(1/x) & x \neq 0 \\
0 & x = 0.
\end{cases}
\]

Show that \( f \) is differentiable, but \( f' \) is not continuous on \( \mathbb{R} \). \textit{Hint:} You need to use the definition of the derivative at \( x = 0 \) to compute \( f'(0) \).

3. For \( n \in \mathbb{Z}_{\geq 0} \), define \( g_n(x) = 2xne^{-nx^2} \) for \( x \in [0, 1] \).

   (a) Show that for each \( x \in [0, 1] \), we have \( \lim_{n \to \infty} g_n(x) = 0 \).

   (b) Using the Fundamental Theorem of Calculus to evaluate the integrals on the left, show that

\[
\lim_{n \to \infty} \int_0^1 g_n(x) \, dx \neq \int_0^1 \left( \lim_{n \to \infty} g_n(x) \right) \, dx.
\]

4. Suppose \( f \in C^2([0, 1]) \), the space of twice differentiable functions with continuous second-derivatives.

   If \( f(0) = f'(1) = 0 \) and \( \int_0^1 f(x)f''(x) \, dx = 0 \), show that \( f \equiv 0 \).