12.) Prove that the following is partially correct with the initial assertion "a and d are positive integers" and the final assertion "q and r are integers such that \( a = dq + r \) and \( 0 \leq r < d \).

Proof: Consider the loop invariant
\[ p = " a = dq + r \), and \( r \geq 0 \)."

When the program executes, we set \( r = a \) and \( q = 0 \), so \( p \) is true because the initial assertion is true, and \( a = d \cdot 0 + r \).

Inside the loop, the first assignment replaces \( r \) with \( r - d \), because \( r \geq d \), we know \( r - d \geq 0 \), so \( r \) remains positive. \( q \) gets replaced with \( q + 1 \), and we know
\[ a = dq + r = d(q+1) + (r-d) \]
so the loop invariant remains true.
Because $r$ is decremented by a positive amount, the loop will terminate.

After termination, we have

\[ \text{Condition} \land p = \neg (r \leq d) \land p \]

\[ d = (r < d) \land p \]

\[ = (r < d) \land \neg \neg a = d \cdot q + r \land r \geq 0 \]

\[ = (r < d) \land \neg \neg a = d \cdot q + r \land r \geq 0 \]

\[ = (0 \leq r < d) \land \neg \neg a = d \cdot q + r \]