1. **Simple Harmonic Oscillator** Let’s forget about gravity in this problem. Suppose a block with mass \( m = 1 \text{kg} \) is attached to a spring with a constant \( k = 4 \text{N/m} \). The block slides horizontally on a surface with no friction.

   (a) Write down a differential equation that models this problem. What does \( x(t) \) represent in your solution? There are many things we could measure, but one makes a lot more sense than others.

   (b) Suppose that at time \( t = 0 \), the velocity is \( v(0) = 1 \text{m/sec} \) and the position \( x(0) = 0 \). Solve the differential equation from part (a).

   (c) Let’s add in some damping. Suppose that in addition, there is a damping constant of \( b = 2 \text{Nm/sec} \). What does the new differential equation look like? Solve the new problem and compute \( \lim_{t \to \infty} x(t) \) if it exists.

   (d) What is the minimum amount of damping that needs to be added in order to get the spring to come to rest? (This is called critically damped - and you get wildly different answers on either side of this point!).

2. Suppose we have a bucket of salt and water. Let \( x(t) = \text{amount of salt in the bucket} \). Suppose that a salt mixture enters the bucket at a rate of \( 2 \text{ L/sec} \) and the concentration of this salt is \( 2 \text{g/L} \). The mixture in the bucket is well mixed, and salt drains from the bucket at the exact same rate. There is initially \( 100 \text{ L of pure water in the bucket} \).

   (a) Write down a formula for the concentration of the salt in the bucket. *Hint:* This part requires very simple algebra!

   (b) What’s the concentration of the salt that leaves the bucket?

   (c) Write down a differential equation that models this problem. It should be of the form

   \[
   \frac{dx}{dt} = \{\text{Stuff in}\} - \{\text{Stuff out}\}.
   \]

   What are the correct units for stuff in and stuff out?

   (d) What are the correct initial conditions for \( x(t) \)?
(e) Find the amount of salt in the bucket. What is $\lim_{t \to \infty} x(t)$? What should this limit be?

3. Use variation of parameters to solve the following problems.

(a) $y'' + 4y = \csc(t)$
(b) $y'' + 9y = 9 \sec^2(3t)$.
(c) $y'' + y = \tan(t)$
(d) $y'' + 4y' + 4y = t^{-2}e^{-2t}$
(e) $y'' - 2y' + y = e^t/(1 + t^2)$

4. Get a head start on your new homework!