1. Find the derivative or anti-derivative of the following functions. For each of these problems, discuss with your group members how you knew what method to try and/or use.

(a) \( \int \frac{\ln(x)}{x} \, dx \)

(b) \( \int \frac{(\ln(x))^3}{x} \, dx \)

(c) \( \int x^3 \sin(x^4) \, dx \)

(d) \( \int_{\pi/2}^{\theta} \theta^2 \sin(2\theta) \, d\theta \)

2. **A Proof that 0 = -1.** Use integration by parts to show that \( \int \tan(x) \, dx = -1 + \int \tan(x) \, dx \). If we subtract \( \int \tan(x) \, dx \) from both sides, we get 0 = -1. Can you recreate this? What just happened!? Is IBP broken? 
**Hint:** Are anti-derivatives unique?

3. Use IBP to find the following anti-derivatives

(a) \( \int \sin(x)e^{2x} \, dx \)

(b) \( \int \sin(3x)e^{2x} \, dx \)

(c) \( \int \sin(x) \cos(2x) \, dx \)

(d) What’s similar and different between parts (a) and (c)? Can you integrate \( \int \sin(x) \sin(mx) \, dx \) for any \( m \)? **Hint:** The case where \( m = 1 \) is a special case

(e) Try \( \int \sin(nx) \sin(mx) \, dx \). What happens when \( n = m \)?

(f) Use your result from (e) to find \( \frac{1}{\pi} \int_{0}^{2\pi} \sin(nx) \sin(mx) \, dx \).

(g) Use your result from (f) to find \( \frac{1}{\pi} \int_{0}^{2\pi} f(x) \sin(nx) \, dx \), where \( f(x) = 2 \sin(x) - 10 \sin(5x) + 50 \sin(13x) \).

4. Suppose \( f(x) \) is a function that satisfies \( \int_{0}^{1} f(x) f''(x) \, dx = 0 \). Show that if \( f(0) = f'(1) = 0 \), then \( f(x) = 0 \) for all \( x \in [0,1] \).

5. There are at least two schools of thought for doing partial fractions. The first involves equating coefficients, and I’ll call the second method the ‘plug and chug’ method. In both cases, you start out by multiplying both sides by the denominator of the left hand side, then proceeding with a method. Use the method of equating coefficients to integrate the following:
(a) \( \int \frac{1}{(x-1)(x+2)} \, dx \)
(b) \( \int \frac{x+2}{x^2-6x+9} \, dx \)
(c) \( \int \frac{x}{(x^2+1)(x-1)} \, dx \)
(d) \( \int \frac{x^3-2x+1}{(x+1)(x-2)^2} \, dx \)
(e) \( \int \frac{x^3-2x+1}{x^2-2x^2+x-2} \, dx \)
(f) \( \int \frac{z-1}{z(z+1)} \, dz \)

Repeat these problems using the other method. Which do you prefer? When is one method easier than the other? Can you combine these methods?

6. The ‘Heaviside’ method can be used for monomial terms in a partial fraction decomposition. Find \( A \) if \( \frac{p(x)}{q(x)(x-r)} = \frac{A}{x-r} + \frac{p_1(x)}{q_2(x)} \). Use this method, and in each case indicate what you’re using for \( p, q \) and \( r \).

(a) \( \frac{x}{(x+1)(x-2)} \)
(b) \( \frac{x^2+1}{x(x-1)(x+5)} \)

How does this method compare with the methods in problem 5?

7. (a) Set \( I_n = \int \cos^n x \, dx \). Show that \( I_n = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \sin^{n-2} x \, dx \).

What values of \( n \) does this hold for? What’s \( I_1 \) and \( I_3 \)? Can you find \( I_3 \) using a different method?
(b) If we set \( I_n = \int (\ln x)^n \, dx \), can you find a formula for \( I_n \) in terms of \( I_{n-1} \)? Find \( I_1 \) by hand, and use that to find \( I_2 \).

8. Integrands oftentimes involve \( \sqrt{a^2-x^2} \), \( \sqrt{x^2-a^2} \) or \( \sqrt{x^2+a^2} \). For each of these cases, can you come up with a useful substitution that may help to simplify an integrand? It may be helpful to start with your favorite trig id: \( \sin^2(\theta) + \cos^2(\theta) = 1 \). Use what you deduced to evaluate the following:

(a) \( \int \frac{1}{\sqrt{1-x^2}} \, dx \)
(b) \( \int \frac{1}{(4-x^2)^{3/2}} \, dx \)
(c) \( \int \frac{2x^3}{\sqrt{x^2+9}} \, dx \)
(d) \( \int \frac{\sqrt{1-(\ln x)^2}}{x \ln x} \, dx \)
(e) \( \int_1^e \frac{dy}{y\sqrt{1+(\ln y)^2}} \)