1. (2 Points) Compute:
\[ \int \frac{1}{x^{7/6} + x} \, dx. \]
*Hint:* \(x^{7/6} + x = x(x^{1/6} + 1).\)

2. (2 Points) Integrate:
\[ \int \frac{d\theta}{\sqrt{1 + \sqrt{\theta}}}. \]

3. (2 Points) Compute:
\[ \int_{-1}^{3} \frac{4x^2 - 7}{2x + 3} \, dx. \]

4. (2 Points) Integrate:
\[ \int \frac{1}{(r + 1)\sqrt{r^2 + 2r}} \, dr. \]

5. (2 Points) Compute:
\[ \int_{\ln(2)}^{\infty} \frac{e^t}{e^{2t} + 3e^t + 2} \, dt. \]

6. (2 Points) The region in the first quadrant that is enclosed by the \(x\)-axis, the curve \(y = 5/x\sqrt{5 - x}\), and the lines \(x = 1\) and \(x = 4\) is revolved about the \(x\)-axis to generate a solid. Find the volume of the solid.

7. (2 Points) Suppose \(n, m \in \{1, 2, 3, \ldots\}\), the set of positive integers. Calculate \(\frac{1}{\pi} \int_0^{2\pi} \sin(mx) \sin(nx) \, dx\). *Hint:* Treat the case \(n = m\) as a special case.

8. (2 Points) Suppose \(\int_{-\infty}^{\infty} f(x) \, dx = 1\) where \(f(x) = C|x|e^{-kx^2}\) for some \(k > 0\). *Hint:* For each of these parts, calculate \(\int_0^{\infty} f(x) \, dx\) and \(\int_{-\infty}^{0} f(x) \, dx\) separately. Add your answers. You may assume (without justification) that \(\int_{0}^{\infty} e^{-x^2} \, dx = \sqrt{\pi}/2\).
(a) Find \( C \).
(b) Find the value \( \mu := \int_{-\infty}^{\infty} xf(x) \, dx \).

9. (4 Points) The **Laplace Transform** is defined as \( \mathcal{L}\{f(t)\}(s) := \int_{0}^{\infty} f(t)e^{-st} \, dt \). It is common to write \( \hat{f}(s) := \mathcal{L}\{f(t)\}(s) \).

(a) If \( f(t) = t \), find \( \hat{f}(s) \), where \( s > 0 \).
(b) If \( f(t) = e^{\alpha t} \), find \( \hat{f}(s) \), where \( s > \alpha \).
(c) If \( f(t) = \sin(\alpha t) \), find \( \hat{f}(s) \), where \( s > 0 \).
(d) If \( f(t) = \cos(\alpha t) \), find \( \hat{f}(s) \), where \( s > 0 \).

Why must we assume these restrictions on \( s \)?
*Hint:* The correct answers you should expect are the following:

(a) \( \frac{1}{s^2} \)
(b) \( \frac{1}{s-\alpha} \)
(c) \( \frac{\alpha}{s^2+\alpha^2} \)
(d) \( \frac{s}{s^2+\alpha^2} \)

Do not be confused about the notation. For part (a), all you need to find is \( \int_{0}^{\infty} te^{-st} \, dt \). Think of \( s \) as a fixed constant for each of the integrals you compute. Parts c and d are essentially the same.
1. (3 Points) Solve this differential equation explicitly for $y$:

$$\frac{dy}{dx} = 3x^2 e^{-y}$$

2. (3 Points) Solve this differential equation explicitly for $y$:

$$x \frac{dy}{dx} + 2y = x^3, \quad x > 0, \quad y(2) = 1.$$
1. (6 Points) Sketch the the region described by $9y^2 \leq 36 + 4x^2$. Include all relevant information: location of foci, vertices, asymptotes/minor vertices (if any). Indicate how you decide where to shade.

2 a. (1 Point) Identify the conic section described by $x^2 + 4y^2 = 3 + 2x$.

2 b. (5 Points) Sketch a graph of the the conic section described by $\frac{(x-1)^2}{4} + y^2 = 1$. Include location of foci and major/minor vertices.

3. (3 Points) Susan decides she wants to buy a telescope and walks into a Wal-Mark. The teller informs her their telescopes are made with hyperbolic mirrors instead of parabolic mirrors. Fortunately, Susan took Professor Isaac’s math 222 course and knew better. Why does Susan leave the store? (You may write your answer on the back - an answer of ‘they don’t work’ will not receive any credit).
Math222-4, Spring 2007
Quiz #7: 03–28–07
No Calculators. You may use one 3 × 5 index card. Due: March 29th, after class.

1. (5 Points) Set up (AND INTEGRATE) an integral equation for the area inside both the curves described by \( r = 1 - \cos(\theta) \) and \( r = \cos(\theta) \).

2. Consider the conic section described by \( r = \frac{1}{1 - \frac{1}{2} \sin(\theta - \frac{\pi}{3})} \).
   a.) (2 Points)
   What is \( e \), the eccentricity? What kind of conic section is it? Locate one of the foci and the two major vertices. (Polar or cartesian coords is fine).
   b.) (3 Points)
   Sketch a graph of this curve.

3. (5 points) For \( f(x) = (1/3)x^3 - x^2 + 6x - 2 \), find the Taylor series for \( f \) centered at \( a = 1 \).

4. (5 Points) Find the MacLauren series for \( f(x) = \frac{x^2}{1+x} \). \( Hint: \) use a series you already know...
1. (5 Points) Determine whether or not the following series converge. If it converges, find its sum.

a.) \[ \sum_{n=2}^{\infty} \frac{1}{(n-1)(n+1)} \].

b.) \[ \sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n \].

c.) \[ \sum_{n=1}^{\infty} \frac{n^n}{n!} \].

d.) \[ \sum_{k=10}^{\infty} \frac{5^{k-5}}{3^{k+1}2^{k-2}} \].

e.) \[ \sum_{n=0}^{\infty} \left(\tan^{-1}(n) - \tan^{-1}(n + 1)\right) \].

2. (3 Points) This problem was taken directly from one of Professor Isaacs' old exams.

a.) State the ‘no way’ \((n^{th})\)-term test.

b.) Suppose I know that the partial sums of a series satisfy \(S_n = 1 - 1/n\) for each natural number \(n\). Does the series converge?

c.) What is the fourth term in the series?

3. (2 Points)

a.) Find the equation of a line through \((2, 3, 0)\) that is perpendicular to both vectors \(\vec{u} = \vec{i} - 2\vec{j} + \vec{k}\) and \(\vec{v} = 3\vec{i} + \vec{j} - 2\vec{k}\).

b.) Find the equation of the plane which passes through the point \((2, 3, 0)\) and is perpendicular to the line from part a.
4. (2 Points) Find the distance from the point \( P(2, 2, 3) \) to the plane \( 2x + y + 2z = 3 \).

5. (2 Points) Use vector algebra to show that squares are the only rectangles with perpendicular diagonals.

6. (2 Points) Determine if the plane \( 2x + 3y - 5z = 7 \) is parallel, perpendicular or neither to the following planes. Justify your reasoning.
   a.) \(-6x - 9y + 15z = 7\).
   b.) \(4x - y + z = 10\).

7. (1 Point) Find the equation of a plane that passes through the origin and the two points \( A(0, 2, 1) \) and \( B(1, -3, 2) \).

8. (2 Points)
   a.) For vectors \( \vec{v} \) and \( \vec{u} \), show that \(|\vec{v} \cdot \vec{u}| \leq |\vec{v}||\vec{u}|\). This is known as the Cauchy-Schwartz inequality. Under what conditions do we have equality?
   
   b.) (Triangle Inequality) Suppose \( \vec{v} \) and \( \vec{u} \) are vectors. Use vector algebra to show that \(|\vec{v} + \vec{u}| \leq |\vec{v}| + |\vec{u}|\). Under what conditions do we have equality? (Hint: Look at the quantity \(|\vec{v} + \vec{u}|^2\).)

9. (1 Point) In the formula \( \text{proj}_{\vec{w}}(\vec{v}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{w}|^2} \vec{w} \), the length of \( \vec{w} \) doesn’t matter. Show that for \( c \neq 0 \), we have \( \text{proj}_{c\vec{w}}(\vec{v}) = \text{proj}_{\vec{w}}(\vec{v}) \).