Directions:

- Print out this piece of paper and use it as a cover sheet. Write your name in the upper right hand corner.
- Your homework should be stapled and each problem should occur in order.
- Do not hand in scratch work.
- Homework is due at the start of class.

1. Problems

2. If \( a_n \) is a sequence with \( \lim_{n \to \infty} a_n = 0 \), prove that \( \lim_{n \to \infty} a_n^2 = 0 \).
3. Suppose \( \{a_n\} \) is a sequence of positive real numbers that converges to \( a \). Prove that \( a \geq 0 \).
   \text{Hint:} \ try a proof by contradiction.
4. Suppose \( \{a_n\} \) is a sequence of positive real numbers, and that \( \lim_{n \to \infty} a_n = a > 0 \). Prove that \( \lim_{n \to \infty} \sqrt{a_n} = \sqrt{a} \). \text{Hint:} \ “irrationalize the denominator.”
5. Define the sequence \( a_1 = 1 \), and \( a_{n+1} = 5a_n^3 \) for all \( n \geq 1 \).
   (a) If \( a = \lim_{n \to \infty} a_n \), exists, prove that \( a = 0 \) or \( a = 1/5 \).
   (b) Does \( a = \lim_{n \to \infty} a_n \) exist?
   (c) Do parts (a) and (b) present a contradiction?
6. Find an example of a convergent sequence \( \{s_n\} \) of irrational numbers that has a rational number as a limit.