Math 221-01, Fall 2011
Quiz #7 : 11–30–11
No notes or calculators may be used for this quiz. The only thing you can use is a writing utensil and that amazing neural network of grey matter in your head.

1. (5 Points) Find the area of the 'triangular' region enclosed by $y = \sin(x)$ and $y = \cos(x)$ contained within the first quadrant. *Hint:* The first time the graphs run into each other is at $x = \pi/4$.

$$A = \int_{\pi/4}^{\pi/4} \cos x - \sin x \, dx$$

$$= \left[ \sin x + \cos x \right]_{0}^{\pi/4}$$

$$= \left( \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right) - \left( 0 + 1 \right)$$

$$= \sqrt{2} - 1$$

2. (5 Points) Find the volume of the solid object created by rotating the region enclosed by the graphs of $y = \frac{1}{8}x^2$ and $y = \sqrt{x}$ about the $x$-axis.

$X = 0 \text{ to } X = 4$ use two washers.

Washers Method:

$$V = \pi \int_{0}^{4} \left( \frac{1}{8} x^2 \right)^2 - \left( \sqrt{x} \right)^2 \, dx = \pi \int_{0}^{4} x - \frac{1}{64} x^4 \, dx$$

$$= \pi \left[ \frac{1}{2} x^2 - \frac{1}{64 \cdot 5} x^5 \right]_{0}^{4} = \pi \left( \frac{4^2}{2} - \frac{4^5}{64 \cdot 5} \right)$$
Math 221-01, Fall 2011
Quiz #8 : 12-6-11
No notes or calculators may be used for this quiz. The only thing you can use is a writing utensil and that amazing neural network of grey matter in your head.

1. (5 Points) Find the volume of the solid constructed by revolving the region bounded by $x = \sqrt{y}$, $x = -y$ and $y = 2$ about the $x$-axis. You may use any method of your choosing.

![Diagram]

**Shell Method:**

$$2\pi \int_0^2 y \left( \sqrt{y} - (-y) \right) \, dy$$

**Disk Method:**

$$\pi \int_0^2 y^2 - (-y)^2 \, dx + \int_0^2 \pi^2 - (x^2)^2 \, dx$$

2. (2 Points) If $x = x(y)$, that is, $x$ is a function of $y$, what is the formula for the arc length of the curve when $a \leq y \leq b$?

$$L = \int_a^b \sqrt{1 + \left( \frac{dx}{dy} \right)^2} \, dy$$

3. (3 Points) Suppose $x = \int_0^y \sqrt{\sec^2 t - 1} \, dt$. Find the length of the curve if $0 \leq y \leq \pi/4$. 

*Hint:* Recall FTC, version I. That is, if $F(z) = \int_a^z f(t) \, dt$, then $F'(z) = f(z)$.

$$\frac{dy}{dy} = \sec^4 y - 1$$

$$1 + \left( \frac{dx}{dy} \right)^2 = \sec^4 y$$

$$L = \int_0^{\pi/4} \sqrt{1 + (x')^2} \, dy = \int_0^{\pi/4} \sec^2 y \, dy$$

$$= \tan y \bigg|_0^{\pi/4} = 1$$
Math 221-01, Fall 2011
Quiz #9: 12–14–11
No notes or calculators may be used for this quiz. The only thing you can use is a writing utensil and that amazing neural network of grey matter in your head.

1. (2 Points) What's the definition of \( \ln(x) \)? *Hint:* It's written in terms of an integral.

\[
\ln x = \int_{1}^{x} \frac{1}{t} \, dt.
\]

2. (3 Points) Compute \( \frac{dy}{dx} \) if \( y = \ln(\ln(x)) \).

\[
\frac{dy}{dx} = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \ln x}
\]

3. (2 Points) Show that \( \frac{d}{dx} e^x = e^x \) by starting with \( y = e^x \), taking logarithms of both sides, and then solving for \( \frac{dy}{dx} \).

\[
y = e^x \quad \Rightarrow \quad \ln y = x \quad \Rightarrow \quad \frac{1}{y} \frac{dy}{dx} = 1
\]

\[
\frac{dy}{dx} = e^x
\]

4. (3 Points) Compute \( \frac{dy}{dx} \) if \( y = \ln(x e^{-x}) \).

\[
y = \ln(x e^{-x}) = \ln x + \ln(e^{-x}) = \ln x - x
\]

\[
y' = \frac{1}{x} - 1. \text{ This is easier than:}
\]

\[
y' = \frac{1}{x e^{-x}} \left( e^{-x} + x (-e^{-x}) \right) = \frac{e^{-x}}{x e^{-x}} = \frac{1}{x} - 1.
\]