An introduction to the cyclic sieving phenomenon

by

Bruce E. Sagan
Department of Mathematics
Michigan State University
East Lansing, MI 48824-1027, USA
sagan@math.msu.edu

Let $S$ be a set which admits an action of the cyclic group $C_n$ of order $n$. Let $f(q)$ be a polynomial in $q$. Usually $f(q)$ will be the generating function for some statistic on $S$. Finally, let $g$ be a generator of $C_n$ and $\gamma = e^{2\pi i/n}$. We say that the pair $(S, f(q))$ exhibits the cyclic sieving phenomenon if, for every integer $d$,

$$f(\gamma^d) = \text{number of element of } S \text{ fixed by } g^d.$$ 

This concept was first introduced and studied by Reiner, Stanton and White as a generalization of Stembridge’s $q = -1$ phenomenon which is the case $n = 2$. Since then it has been the subject of a growing number of papers. This talk will serve as an introduction to this area of research.