Directions: Please write your solutions to the following problems neatly and carefully. Write your solutions on looseleaf paper. Each of your solutions will be graded according to three criteria: correctness, clarity, and completeness. Each problem MUST begin with a complete statement of the problem.

Note: The purpose of the homework problems is two fold. First I would like to see and read how you communicate mathematics so that I can help you improve in this area. Second I think that it is important for your long term learning and success in mathematics to regularly think about problems which go beyond direct computation. However, as you will discover, problems which go beyond direct computation can be challenging and time consuming.

Due Date: Friday, 10/24, at the start of class.

1. Write the complete statement of problem #12 on the “Extrema on an Interval” WebAssign assignment. (Your problem may be different than other students since many WebAssign problems have the numerical values randomized. Be sure to solve the problem which has been assigned to you.) Then write a detailed solution to the problem. Please write a sentence or two which explains why your answer is correct, or if you are not sure it is correct, explain why you feel this way.

2. Suppose the graph below is the graph of the curve $y = f'(x)$. Using this graph describe the intervals on which the function $f(x)$ is increasing. Also, describe the intervals on which the function $f(x)$ is concave down. Finally, sketch an approximate graph of the function $f(x)$. Assume that $f(0) = 0$. 
3. The concept of taking the limit of a function $f(x)$ as $x$ tends to infinity allows us to make precise statements like

$$\sqrt{x + 10} \approx \sqrt{x}$$

for $x$ sufficiently large.

The precise definition is as follows: we say that two functions $f(x)$ and $g(x)$ are approximately equal for sufficiently large values of $x$ if

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = 1.$$ 

The above relationship between $f(x)$ and $g(x)$ is often expressed by the following symbols: $f(x) \sim g(x)$. Which of the following statements are true? Justify each answer by computing an appropriate limit.

(a) $\sqrt{x + 10} \sim \sqrt{x}$
(b) $x^2 + 30x - 27 \sim 3x^2 + 5$
(c) $\sin x \sim x$
(d) $\tan^{-1} x \sim \frac{x}{2}$
(e) $x \ln (x^2) \sim (2x + 1) \ln x$.

Note: Please read sections 4.5 and 4.6 for examples of how to compute limits at infinity and how to write your solutions.
4. This builds on the last problem. Please read the first two sections of the Wikipedia page on the “Prime number theorem”. Then answer the following questions.

(a) What are the first 20 positive prime numbers? (Important: The number 1 is by definition not a prime number.)
(b) Approximately how many integers between one and one billion are prime?
(c) Approximately how many numbers between one billion and ten billion are prime?
(d) What approximately is the probability that a number between one billion and ten billion is prime?