Directions: Please write your solutions to the following problems neatly and carefully. Write your solutions on looseleaf paper. Each of your solutions will be graded according to three criteria: correctness, clarity, and completeness. Each problem MUST begin with a complete statement of the problem.

Note: The purpose of the homework problems is two fold. First I would like to see and read how you communicate mathematics so that I can help you improve in this area. Second I think that it is important for your long term learning and success in mathematics to regularly think about problems which go beyond direct computation. However, as you will discover, problems which go beyond direct computation can be challenging and time consuming.

Due Date: Monday, 09/15, at the start of class.

1. Write the complete statement of problem #10 on the “Definition of a Limit” WebAssign problem. (Your problem may be different than other students since many WebAssign problems have the numerical values randomized. Be sure to solve your problem.) Then write a detailed solution to the problem. Below each part, please write a sentence or two which explains why your answer is correct, or if you are not sure it is correct, explain why you feel this way.

2. Using the same instructions as above (state the problem, then solve it, then write a sentence or two explaining your reasoning or explaining why you are unsure of your answer), solve problem #16 on the “Definition of a Limit” WebAssign.

3. Using the same instructions as above (state the problem, then solve it, then write a sentence or two explaining your reasoning or explaining why you are unsure of your answer), solve problem #11 on the “Computation of Limits, Part 2” WebAssign.

4. In this problem, we will study the properties of limits in Theorem 2.2 of section 2.3. Compute each of the following limits and clearly indicate which properties you are using at each step of your solution. Here is an example of the level of detail I would like see:

Problem:

\[
\lim_{x \to 0} \frac{x^2 - x}{\sin x}.
\]
Solution:

\[
\frac{x^2 - x}{\sin x} = \frac{x(x - 1)}{\sin x} = \frac{x}{\sin x} \cdot (x - 1)
\]

Since \( \lim_{x \to 0} \frac{\sin x}{x} = 1 \) (by Theorem 2.9.1), we can apply Theorem 2.2.4 (quotient) to deduce that

\[
\lim_{x \to 0} \frac{x}{\sin x} = \lim_{x \to 0} \frac{1}{\sin x} = \frac{1}{1} = 1.
\]

Using Theorem 2.2.2 (sum), we see that

\[
\lim_{x \to 0} (x - 1) = \lim_{x \to 0} x + \lim_{x \to 0} -1 = 0 - 1 = -1,
\]

where the last two limits follow from Theorems 2.1.1 and 2.1.2.

Finally, using Theorem 2.2.3 (product), we see that

\[
\lim_{x \to 0} \frac{\sin x}{x} \cdot (x - 1) = 1 \cdot -1 = -1.
\]

Now write equally detailed solutions to each of the following problems.

(a)

\[
\lim_{x \to -1} \frac{x^2 + 2x + 1}{x^2 + 3x + 2}
\]

(b)

\[
\lim_{x \to 0} \frac{1 - \cos x}{\sin x}
\]