Section 2.1

2. A (not in the text) Which of the following are statements? Explain.

1. Let $x$ be a positive integer. Then $\sqrt{x}$ is rational.

2. Mathematics is fun.

3. The President of the United States in 2089 will be a woman.

4. The integer 105 is prime.

2.2 Consider the sets $A, B, C,$ and $D$ below. Which of the following statements are true? Give an explanation for each false statement.

- $A = \{1, 4, 7, 10, 13, 16, \ldots \}$, $C = \{x \in \mathbb{Z} \mid x$ is prime and $x \neq 2\}$,
- $B = \{x \in \mathbb{Z} \mid x$ is odd\}, $D = \{1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots \}$

(a) $25 \in A$

(b) $33 \in D$

(c) $22 \notin A \cup D$

(d) $C \subseteq B$

(e) $\emptyset \in B \cap D$

(f) $53 \notin C$

2.4 Consider the open sentence $P(x) : x(x - 1) = 6$ over the domain $\mathbb{R}$.

(a) For what values of $x$ is $P(x)$ a true statement?

(b) For what values of $x$ is $P(x)$ a false statement?

Section 2.2

2.14 State the negation of each of the following statements.

(a) At least two of my library books are overdue.

(b) One of my two friends misplaced his homework assignment.

(c) No one expected that to happen.

(d) It’s not often that my instructor teaches that course.
(e) It’s surprising that two students received the same exam score.

Section 2.3

2.16 (2.11 in 2nd edition) For the sets $A = \{1, 2, \ldots, 10\}$ and $B = \{2, 4, 6, 9, 12, 25\}$ consider the following two statements:

$$P : A \subseteq B, \quad Q : |A - B| = 6.$$ 

Determine which of the following statements are true.

(a) $P \lor Q$
(b) $P \lor (\sim Q)$
(c) $P \land Q$
(d) $(\sim P) \land Q$
(e) $(\sim P) \lor (\sim Q)$

2.18 (2.13 in 2nd edition) Let $S = \{1, 2, \ldots, 6\}$ and let

$$P(A) : A \cap \{2, 4, 6\} = \emptyset, \quad Q(A) : A \neq \emptyset$$

be open sentences over the domain $\mathcal{P}(S)$.

(a) Determine all $A \in \mathcal{P}(S)$ for which $P(A) \land Q(A)$ is true.
(b) Determine all $A \in \mathcal{P}(S)$ for which $P(A) \lor (\sim Q(A))$ is true.
(c) Determine all $A \in \mathcal{P}(S)$ for which $(\sim P(A)) \land (\sim Q(A))$ is true.