Section 10.1

10.1 Let $S = \{A_1, A_2, \ldots, A_5\}$ be a collection of five subsets of the set $A = \{-5, -4, \ldots, 4, 5\}$, where

- $A_1 = \{x \in A \mid 1 < x^2 < 10\}$
- $A_2 = \{x \in A \mid (x + 2)(x - 4) > 0\}$
- $A_3 = \{x \in A \mid |x + 2| + |x - 3| \leq 5\}$
- $A_4 = \{x \in A \mid \frac{1}{x^2 + 1} > \frac{2}{5}\}$
- $A_5 = \{x \in A \mid \sin\left(\frac{\pi x}{4}\right) = 0\}$

Determine the cardinality of each set above. Which of these sets are numerically equivalent? Prove your assertions.

Section 10.2

10.5 Prove that $|\mathbb{Z} - \{2\}| = |\mathbb{Z}|$.

10.6 (a) Prove that the function $f : \mathbb{R} - \{1\} \rightarrow \mathbb{R} - \{2\}$ defined by $f(x) = \frac{2x}{x - 1}$ is a bijection.

(b) It follows from (a) that $|\mathbb{R} - \{1\}| = |\mathbb{R} - \{2\}|$. Explain why this is so.

(Bonus) Assume that $A$ and $B$ are nonempty sets and that $|A| = |B|$. Let $a \in A$ and $b \in B$. Prove that $|A - \{a\}| = |B - \{b\}|$.

10.8 Prove that the function $f : \mathbb{N} \rightarrow \mathbb{Z}$ defined by $f(n) = \frac{1}{4}(1 + (-1)^n(2n - 1))$ is a bijection.