Please write your solutions on a different piece of paper. Please refer to the course syllabus for a detailed explanation of how you should write homework solutions and how they will be graded.

Section 4.1

4.2 (4.2 in second edition) Let \( a, b \in \mathbb{Z} \), where \( a \neq 0 \) and \( b \neq 0 \). Prove that if \( a | b \) and \( b | a \), then \( a = b \) or \( a = -b \).

4.4 (4.4 in second edition) Let \( x, y \in \mathbb{Z} \). Prove that if \( 3 \nmid x \) and \( 3 \nmid y \), then \( 3 | (x^2 - y^2) \).

4.6 (4.6 in second edition) Let \( a \in \mathbb{Z} \). Prove that if \( 3 | 2a \), then \( 3 | a \).

Section 4.2

4.14 (4.10 in second edition) Let \( a, b, n \in \mathbb{Z} \), where \( n \geq 2 \). Prove that if \( a \equiv b \pmod{n} \), then \( a^2 \equiv b^2 \pmod{n} \).

4.18 (not in second edition) Let \( m, n \in \mathbb{N} \) such that \( m \geq 2 \) and \( m | n \). Prove that if \( a \) and \( b \) are integers such that \( a \equiv b \pmod{n} \), then \( a \equiv b \pmod{m} \).

4.22 (4.16 in second edition) Let \( n \in \mathbb{Z} \). Prove each of the following statements.

(a) If \( n \equiv 0 \pmod{7} \), then \( n^2 \equiv 0 \pmod{7} \).

(b) If \( n \equiv 1 \pmod{7} \), then \( n^2 \equiv 1 \pmod{7} \).

(c) If \( n \equiv 2 \pmod{7} \), then \( n^2 \equiv 4 \pmod{7} \).

(d) If \( n \equiv 3 \pmod{7} \), then \( n^2 \equiv 2 \pmod{7} \).

(e) For each integer \( n \), \( n^2 \equiv (7 - n)^2 \pmod{7} \).

(f) For every integer \( n \), \( n^2 \) is congruent to exactly one of 0, 1, 2, or 4 modulo 7.