MTH 254H Homework 5 (due Friday, 02/24/12)

Directions. Please work on the problems below. Your solutions must begin with a clear statement of the problem, followed by a clear, legible solution (or partial progress towards a solution). Please refer to the syllabus for the policy on grading and late homework.

Collaboration. I encourage you to discuss the homework problems with your classmates. However, each student must submit his or her own homework solutions.

- Section 2.2 # 12 – 14
- Section 3.2 # 10, 12, 15
- Section 3.3 # 1 – 5
- Suppose that $f : U \subset \mathbb{R}^2 \rightarrow \mathbb{R}$ is expressed in rectangular coordinates by $f(x, y)$.

1. The expression of $f$ in polar coordinates is given by writing $x = r \cos \theta$ and $y = r \sin \theta$, so that $f$ may be viewed as a function of $(r, \theta)$. Determine an expression for $f_x$ and $f_y$ in terms of $f_r$ and $f_\theta$. And determine an expression for $f_r$ and $f_\theta$ in terms of $f_x$ and $f_y$.

2. Show that Laplace’s equation $f_{xx} + f_{yy} = 0$ has the following expression in polar coordinates:

   $$\frac{\partial^2 f}{\partial r^2} + \frac{1}{r} \frac{\partial f}{\partial r} + \frac{1}{r^2} \frac{\partial^2 f}{\partial \theta^2} = 0.$$

- Suppose that $f, g : U \subset \mathbb{R} \rightarrow \mathbb{R}^3$. Let $h(t) = f(t) \times g(t)$, where $\times$ is the cross product (a.k.a. vector product). Determine a rule which relates the derivative of $h(t)$ to the derivatives of $f(t)$ and $g(t)$. Prove your assertion.