1. (4 points) Determine the inverse function, \( f^{-1}(x) \), of the function \( f(x) \) shown below.

\[ f(x) = \frac{2x - 3}{x + 7} \]

Let \( y = \frac{2x - 3}{x + 7} \). Switch \( x \) and \( y \) and re-write: \( x = \frac{2y - 3}{y + 7} \).

Solve for \( y \) by multiplying by the LCD \((y + 7)\):

\[ x(y + 7) = (2y - 3), \]

then distribute the \( x \) as follows: \( xy + 7x = 2y - 3 \).

Next, gather all the \( y \) terms on one side and put all other terms on the other side:

\[ xy - 2y = -7x - 3, \]

and then factor out \( y \) to obtain the following: \( y(x - 2) = -7x - 3 \). The last algebraic step is to divide both sides by \((x - 2)\):

\[ y = \frac{-7x - 3}{x - 2}. \]

Finally, replace \( y \) by \( f^{-1}(x) \):

\[ f^{-1}(x) = \frac{-7x - 3}{x - 2}. \]
2. (4 points) Write each expression below in the form \( x^m y^n \) for some integers \( m \) and \( n \). Recall that the set of integers is \( \{\ldots, -2, -1, 0, 1, 2, \ldots \} \).

(a) \( \frac{x^2 x^3}{y^4} \)

Use the property \( a^k a^l = a^{k+l} \) to combine the \( x \)-terms in the numerator: \( x^2 x^3 = x^{2+3} = x^5 \).

Re-write the denominator using a negative exponent: \( \frac{1}{y^4} = y^{-4} \)

Combining the above, we have that

\[
\frac{x^2 x^3}{y^4} = x^5 \frac{1}{y^4} = x^5 y^{-4}.
\]

(b) \( \frac{(x^3 y^5)^2}{x y^4} \)

Use the property \( (ab)^k = a^k b^k \) to expand the numerator: \( (x^3 y^5)^2 = (x^3)^2 (y^5)^2 \).

Then use the property that \( (a^k)^l = a^{kl} \) to write each factor of the numerator using a single exponent: \( (x^3)^2 (y^5)^2 = x^{3(2)} y^{5(2)} = x^6 y^{10} \).

Re-write the denominator using a negative exponent: \( \frac{1}{x y^4} = (x y^4)^{-1} \).

Then, as in the previous steps, use the properties \( (ab)^k = a^k b^k \)

and \( (a^k)^l = a^{kl} \) to write each factor using a single exponent:

\[
(xy^4)^{-1} = x^{-1}(y^4)^{-1} = x^{-1} y^{4(-1)} = x^{-1} y^{-4}.
\]

Combining the above and using the property \( a^k b^l = a^{k+l} \), we have that

\[
\frac{(x^3 y^5)^2}{x y^4} = \frac{x^6 y^{10}}{x y^4} = x^6 y^{10} \frac{1}{x y^4} = (x^6 y^{10})(x^{-1} y^{-4}) = x^{6-1} y^{10-4} = x^5 y^6.
\]
3. (2 points) The graph of $f(x)$ is shown below.

(a) What is $f^{-1}(1)$?

Since $f(0) = 1$ (as we can see by looking at the graph), we have that $f^{-1}(1) = 0$ (by definition of the inverse function).

(b) If the domain of $f(x)$ is $[-1, 2]$, what is the domain of $f^{-1}(x)$?

The domain of $f$ is the range of $f^{-1}$. And the range of $f$ is the domain of $f^{-1}$. So, to answer this question, we need to determine the range of $f$ given that its domain is $[-1, 2]$. Since the domain is $[-1, 2]$, the graph shows all of the values of $f(x)$– the graph does not extend to the left or right in the figure above.

Therefore, the range of $f$, which is the $y$-values of points on the graph, can be seen to be those $y$-values from 0.5 up to 4. Since the endpoints are included in $[-1, 2]$, we also include these endpoints in the range of $y$-values. So, the range of $f$ is $[0.5, 4]$.

Therefore, the domain of $f^{-1}$ is $[0.5, 4]$. 