MTH 103 Review Problems for Midterm Exam I

1. (1.1 #16) What are the restrictions on \( x \) in \( \frac{1}{x^2 - 3x - 4} = \frac{5}{x} + 2 \). Answer: \( x \neq -1, 0, 4 \).

2. (1.1 #56) Solve for \( x \): \( \frac{3}{x-2} + \frac{1}{x} = \frac{3}{x-2} \). Answer: no solutions.

3. (1.1 #68) Solve for \( a \): \( \frac{6}{2a-6} - \frac{3}{3-3a} = \frac{1}{a^2-4a+3} \). Answer: \( a = 7/4 \).

4. (1.1 #78) Solve for \( \mu \): \( z = x - \mu \sigma \). Answer: \( \mu = x - \sigma z \).

5. (1.1 #82) Solve for \( f \): \( \frac{1}{f} = \frac{1}{p} + \frac{1}{q} \). Answer: \( f = pq/(p+q) \).

6. (1.1 #90, modified) Solve for \( r \): \( S = \frac{a - Lr}{1 - r} \). Answer: \( r = (S-a)/(S-L) \).

7. (1.2 #8) Courtney took four tests in science class. On each successive test her score improved by 3 points. If her mean score was 69.5%, then what did she score on the first test? Answer: 65%.

8. (1.2 #14, modified) A NHL hockey rink is 115 feet longer than it is wide. If the perimeter is 570 feet, find the dimensions of the rink. (Assume the rink is a rectangle.) Answer: 200 ft \( \times \) 85 ft.

9. (1.2 #36) A small hose can fill a swimming pool in 3 days. A large hose can fill the same pool in 2 days. If both hoses are used, how long will it take to fill the pool? In other words, it takes 1 day, 4 hours, and 48 minutes.

10. (1.2 #56) A bank robber leaves town at 70 miles per hour. Ten minutes later, a police car chases after the robber at 78 miles per hour. How long, after the robber left town, does the police car catch up to the robber? Answer: 13/8 hours; in other words, 1 hour, 37 minutes, and 30 seconds.

11. (1.3 #48) Solve the following equation by completing the square: \( x^2 + 10x + 21 = 0 \). Answer: \( x = -5 \pm \sqrt{16} \).

12. (1.3 #58) Solve the following equation by completing the square: \( 2x^2 + 5x = 14 \). Answer: \( x = -\frac{5}{4} \pm \sqrt{\frac{137}{16}} \).

13. (1.3 #78) Solve for \( x \): \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \). Answer: \( x = \pm a \sqrt{1 + \frac{y^2}{b^2}} \) or \( x = \pm \frac{a}{b} \sqrt{a^2 + y^2} \).

14. (1.3 #84) Use the discriminant to determine the number and type of the roots of the equation \( -x^2 + 2x = 21 \). Answer: \( \Delta = b^2 - 4ac = -80 < 0 \) so there are two complex roots.
15. (1.3 #104) Solve for $x$: \[ \frac{1}{x-1} + \frac{1}{x-4} = \frac{5}{4}. \] Answer: $x = 5, 8/5$.

16. (1.4 #14) The base of a triangle is half as long as its height. If the area of the triangle is 100 square yards, find its height. Answer: 20 yards.

17. (1.4 #24) The height of an object tossed upward with an initial velocity of 104 feet per second is given by the formula \[ h = -16t^2 + 104t, \] where $h$ is the height in feet $t$ seconds after the object is released. Find the time required for the object to return to its point of departure. Answer: 6.5 seconds.

18. (1.4 #36, modified) A merchant could sell his collection of identical digital cameras at list price for a total of $180. If he had three more cameras, he could sell each camera for $10 less and still receive $180 for the entire collection. Find the list price of the cameras and determine how many cameras the merchant had originally. Answer: $30 is the list price, and originally there were six cameras.

19. (1.5 #10) Simplify the complex number: $\sqrt{-225}$. Answer: $-15i$.

20. (1.5 #36) Simplify the complex number: $(3 - 4i)^2$. Answer: $-7 - 24i$.

21. (1.5 #40) Simplify the complex number: $(12 - \sqrt{-4})(-7 + \sqrt{-25})$. Answer: $-74 + 74i$.

22. (1.5 #52) Simplify the complex number: \[ \frac{34 + 2i}{2 - 4i}. \] Answer: $3 + 7i$.

23. (1.5 #84) Simplify the real number: \[ \left| \frac{3}{3 + i} \right|. \] (Recall that $|a + bi| = \sqrt{a^2 + b^2}$.) Answer: $3\sqrt{10}/10$.

24. (1.5 #112) Use the quadratic equation to determine the complex solutions to $a^2 + 4a + 8 = 0$. Answer: $a = -2 \pm 2i$.

25. (1.6 #8) Solve by factoring: $8b^3 - 10b^2 + 3b$. Answer: $b(4b - 3)(2b - 1)$ so $b = 0, 3/4, 1/2$.

26. (1.6 #24) Solve $3r - r^{1/2} - 2 = 0$. Answer: $r = 1$.

27. (1.6 #32) Find all real solutions: $\sqrt{x^2 + 1} = \frac{\sqrt{-7x + 11}}{\sqrt{6}}$. Answer: $x = -5/3, 1/2$.

28. (1.6 #50) Find all real solutions: $\sqrt{8x^3 - 37} = 2x - 1$. Answer: $x = -3/2, 2$.

29. (1.6 #54) Find all real solutions: $\sqrt{x + 2} + 1 = \sqrt{2x + 5}$. Answer: $x = \pm 2$. 