Activity: The Distributive Law and Factoring

The Distributive Law: If $a, b,$ and $c$ are real numbers, then

$$(a + b)c = ac + bc$$

and

$$a(b + c) = ab + ac.$$ 

Instructions: Use the distributive law to expand each expression below. You will need to work with exponents. Below is a reminder of the most important rules.

Exponent Rules: If $a$ is a real number and $m$ and $n$ are non-negative integers, then

$$a^m a^n = a^{m+n}$$

and

$$(a^m)^n = a^{mn}.$$ 

This should agree with your intuition that

$$a^1 = a, \quad a^2 = a \cdot a, \quad a^3 = a \cdot a \cdot a, \quad \text{etc.}$$

1. $(1 + x)y$

2. $(2 + x)(3 + x)$

3. $(1 + x)(y + z)$

4. $(2 + x + x^2)(1 - x)$

5. $(x + y)^2$
6. \((x + 1)^3\)

7. \((x + y + 1)^2\)

Factoring

Why do we factor? One reason that we factor is that an equation such as \(x^2 - 2x + 3 = 0\) can be solved by factoring as \((x - 3)(x + 1) = 0\) and then appealing to the fact that if \(a\) and \(b\) are real numbers and \(ab = 0\), then either \(a = 0\) or \(b = 0\). So, in the example, either \(x - 3 = 0\) or \(x + 1 = 0\); in other words, the solutions are \(x = -1, 3\).

Instructions: Factor each positive integer into prime factors. Factor each quadratic into linear factors.

1. 42
2. 168
3. 2013
4. \(x^2 + 5x + 6\)
5. \(x^2 - x - 56\)
6. \(x^2 - 9\)
7. \(a^2 - b^2\)
8. \(2x^2 + 5x - 3\)