Chapter 11 Review, p. 829

1: How do you compute the component form of $\overrightarrow{PQ}$ given coordinates for $P$ and $Q$? How are vector addition, scalar multiplication, and magnitude defined? What is a unit vector?

3: How do you determine the component form of a vector in $\mathbb{R}^2$ given the magnitude and an angle with the $x$-axis.

15: How can you determine if three points in $\mathbb{R}^3$ are collinear?

21: How can you determine if two vectors are orthogonal, parallel, or neither?

25: How do you use the physicists definition of the scalar product to measure an angle between two vectors in $\mathbb{R}^3$?

31: How do you determine the scalar and vector projections of $\vec{z}$ onto $\vec{x}$?

35, 37, 38: How do you use the vector product to calculate areas and volumes?

39: What is the definition of torque?

43-46: How do you write a vector equation of a line (or, equivalently, a set of parametric equations of a line) in $\mathbb{R}^3$?

47-50: How do you write an equation of a plane in $\mathbb{R}^3$?

55-64: How do you determine the shape of a surface given by an equation in $x$, $y$, and $z$? What are some strategies for graphing such a surface?

69, 71, 73: How are cylindrical and spherical coordinates defined?

Chapter 12 Review, p. 881

7, 8: How do you sketch the trace of a vector-valued function?

15, 17: How do you write parametric equations for a given collection of curves?


31: How do you integrate a vector-valued function?

35: How do you solve an initial value problem involving vector-valued functions?

37: What is the definition of the velocity, speed, and acceleration of a particle whose position is given as a vector-valued function?

47, 53: How do you determine a unit tangent vector at a point? How do you determine a function which computes the unit tangent vector? What interesting geometric property does the derivative of the unit tangent vector function satisfy?
55: How do you write parametric equations for the tangent line to a space curve at a given point?

65: What is the definition of the length of a space curve?

Chapter 13 Review, p. 978

1,2: How do you describe the level surfaces of a function \( f(x, y, z) \)?

3: What is the effect of transformations such as \( f(x, y) + 2 \) or \( f(x, y - 2) \) on the graph \( f(x, y) \)?

5,7: How do you sketch level curves? (Contrary to the statement of this exercise, which suggests using a computer, these functions are simple enough to sketch level curves by hand.)

14: As with functions of one variable, limits can fail to exist. What is a new technique you’ve learned for showing that a limit of a function of two variables fails to exist?

15-24, 27-30: Partial differentiation is a fundamental skill that you need to be able to demonstrate frequently and without error. What does the numerical value of a partial derivative represent?

25: How do you visualize the graph of a function such that \( f_x < 0 \) and \( f_y < 0 \)? Try this though exercise with other sign combinations or by placing conditions on the second partial derivatives.

31,33: How do you verify that a given function satisfies a given partial differential equation (e.g. Laplace’s equation, the wave equation, or the heat equation)?

37: What is the relationship between linear approximation, differentials, and the equation of a tangent plane to the graph of \( z = f(x, y) \)?

41-44: What are some examples of chain rules for functions of several variables? How are these rules used? What are they useful for?

47-50: What is a directional derivative? How do you compute one?

51-54: What is the gradient of a function? What is the significance of the gradient vector of a function \( f(x, y) \) at a point \( (x_0, y_0) \)?

57, 59: How do you determine the equation of a plane tangent to a surface? What if the surface is not given as a function, but rather is given implicitly? For example, what is an equation for the tangent plane to the surface \( x + \sqrt{x} + y + \sqrt{y} + z + \sqrt{z} = 6 \) at the point \((1, 1, 1)\)? Hint: This is a level surface of a function.

65,67: How do you find the relative extrema and saddle points of a function \( f(x, y) \)?

69,70: Can you recognize relative extrema and saddle points by looking at a contour plot?

71, 74: How do you determine the global maximum or minimum value of a
function of two variables? (Careful with the boundary conditions: it should be clear that the optimal values do not occur on the boundary or “at infinity”.)

In section 13.8, exercises 45-54, p. 961, you are asked to determine the global maximum or minimum values of a function of two variables defined on a region with a boundary. How do you solve such a problem?

**Chapter 14 Review, p. 1052**

3-6: What is a double integral? How do you sketch the region of integration given the limits of integration? How can you tell if you should change to polar coordinates?

7: You could use a double integral to compute the area of a triangle? Could you also use a single integral? Could you use vectors?

17: You can use a triple integral to compute the volume of a solid bounded by graphs of functions of \((x, y)\). You could also use a double integral. Try both methods on this problem.

19: How do you compute the average value of a function defined over a region? Bonus: Is there a mean value theorem for double integrals? What would be the statement of such a theorem?

27-30: Have you exercised your brain by thinking about the true / false questions at the end of each section of chapters 11-15? Exercise 28 is pretty devious. It’s false. Can you think of a counterexample?

31: How do you convert a double integral into polar coordinates?

35, 36: What are some strategies for setting up a double or triple integral which computes the volume of a solid which is described geometrically or algebraically?

43: How do you determine the surface area of a surface which is the graph of a function of \((x, y)\)? How does this compare with the method of surface integrals discussed in 15.5 and 15.6? What are some advantages of the methods of chapter 15?

49-52: Integration is tricky. Which of the following tricks will work on these integrals? Tricks: change coordinates, change the order of integration, interpret the integral as the volume or area of a familiar geometric object, interpret the integral as a moment (as used in centers of mass), use symmetry to simplify a calculation, recognize functions as being even or odd (or neither).

If you didn’t find a use for many of these tricks on this set of integrals, can you find ones in exercises in the sections of chapter 14 where they do apply? Try skimming the exercise sets.

55, 56: Would you rather use cylindrical or spherical coordinates for these problems? Try to setup integrals for both.

67-70: What is a Jacobian determinant? What is its significance? How do you apply the change of variables formula?
Chapter 15 Review, p. 1138

1,3: How does one sketch a vector field? What is a gradient vector field? Do such vector fields have another name? Do they enjoy any nice properties?

5-12: How can you test whether a vector field is conservative? Does this test always work? If not, under what conditions does it work? Is there a method for finding a potential function?

13, 15, 19: How do you compute the divergence or curl of a vector field? One of these quantities is again a vector field and the other is a scalar value function: which is which? What is the physical interpretation of divergence and curl?

21, 23, 25: How do you compute a line integral? What if you’re not given a parametrization? Are there any shortcuts you can use? What is the relationship between $\mathbf{F} \cdot \mathbf{T} \, ds$, $\mathbf{F} \cdot d\mathbf{r}$, and $M \, dx + N \, dy + P \, dz$, where $\mathbf{F}$ is as below?

$$\mathbf{F}(x, y, z) = M(x, y, z) \hat{i} + N(x, y, z) \hat{j} + P(x, y, z) \hat{k}$$

31-36: Is there a sneaky way to compute any of these line integrals?

39: What is the definition of work?

41: What is the fundamental theorem of line integrals? Could you state this theorem accurately without needing to look it up in the textbook?

45-50: What is the statement of Green’s Theorem? How is it applied?

54: What is a surface integral? How is one computed?

57, 58: What is the statement of the Divergence Theorem? What is its connection to the problem of computing the flux of a vector field through a surface? How is the theorem applied?

59, 60: What is the statement of Stokes’s Theorem? How is it similar to Green’s Theorem? How is the theorem applied?