Dear SMC:

I was referred to you by my colleague and competitor Mr. Ty N. Gull of T&E Simulations. I am hoping to interest you in the following problem related to game design. If you choose to take the job, I ask that you submit a complete written report by Friday, March 22.

The problem is as follows: suppose that a game is played on a board $B$ which consists, for instance, of a contiguous collection of squares which are orthogonally adjacent. So, for instance, $B$ might be the familiar $8 \times 8$ chess board or it might be a diamond shaped board or it might be a coarse approximation to a circular board. If I choose a specific square, say $s_0$, and regard this as the position of a piece on the board $B$, then I can assign to any other square, say $s_1$, a positive integer equal to the least number of moves required to move from $s_0$ to $s_1$. The rules for movement may vary from game to game, but in the simplest scenario we may assume that moves are by orthogonally adjacent squares. So, for instance, with this restriction, diagonal moves are not legal.

I would like to understand some mathematical aspects of such games. Specifically, I would like to know the average number of moves it takes to move from $s_0$ to some other square $s$. I would like to put together a study of these averages for various board shapes. If you have time, I would also like to read a study of these averages when diagonal (or other more exotic) moves are allowed. Finally, it would be intriguing to understand the average number of moves from a region $R_0$, i.e. a continuous collection of squares, to any other square. You might imagine this last scenario as one in which some region of the board is special and it is valuable to the player to be able to move pieces quickly to this region.

In studying this problem, there are two approaches that come to mind. In its stated form, this is a discrete problem. But I would also like your report to see if continuous methods given any insight. For instance, one might...
average certain distance functions over regions in the plane much as one
averages functions $f(x, y)$ over regions $R$ in the plane in a third semester
calculus class. Those were fun times at the university! But, that was so
long ago. I really need a refresher, and I’m sure that you are the right team
for the job.

Thank you for giving this problem careful consideration. I look forward to
your reply.

Victor E. Player
Senior Scientist
Advanced Game Design