Section 11.2, # 36: Show that \( f(x, y) = \frac{x^4}{x^4 + y^2} \) has no limit as \((x, y) \to (0, 0)\).

If \( x = 0 \) and \( y \to 0 \), then the limit is zero. If \( y = 0 \) and \( x \to 0 \), then the limit is one. Therefore, the limit does not exist.

Section 11.2, # 38: Show that \( f(x, y) = \frac{xy}{|xy|} \) has no limit as \((x, y) \to (0, 0)\).

If \( y = x \) and \( x \to 0^+ \), then the limit is one. If \( y = -x \) and \( x \to 0^+ \), then the limit is negative one. Therefore, the limit does not exist.

Section 11.2, # 44: If \( f(x_0, y_0) = 3 \), what can you say about the limit of \( f(x, y) \) as \((x, y) \to (x_0, y_0)\) if \( f \) is continuous at \((x_0, y_0)\)? If \( f \) is not continuous at \((x_0, y_0)\)?

If \( f \) is continuous at the point, then by the definition of continuity (see p. 594), the limit exists and is equal to \( f(x_0, y_0) \), which is equal to 3. If \( f \) is not continuous it means that either the limit does not exist or, if it does exist, it is not equal to 3. For example, \( f(x, y) = 0 \) if \( x < 0 \) and \( f(x, y) = 3 \) if \( x \geq 0 \) is not continuous at \((0, 0)\) because the limit of \( f \) as \((x, y) \to (0, 0)\) does not exist. On the other hand if \( f(x, y) = 0 \) if \((x, y) \neq (0, 0)\) and \( f(0, 0) = 3 \), then the limit of \( f \) as \((x, y) \to (0, 0)\) exists, but it is equal to 0, which is not equal to \( f(0, 0) = 3 \).

Section 11.3, # 26: Determine the partial derivatives of

\[
 f(x, y, z) = (x^2 + y^2 + z^2)^{-1/2}
\]

\[
 f_x = (-1/2)(x^2 + y^2 + z^2)^{-3/2}(2x)
\]

\[
 f_y = (-1/2)(x^2 + y^2 + z^2)^{-3/2}(2y)
\]

\[
 f_z = (-1/2)(x^2 + y^2 + z^2)^{-3/2}(2z)
\]

Section 11.3, # 30: Determine the partial derivatives of

\[
 f(x, y, z) = yz \ln(xy)
\]
\[ f_x = yz(1/xy)(y) = yz/x \]

\[ f_y = z \ln(xy) + yz(1/xy)(x) = z \ln(xy) + z \text{ (product rule was used)} \]

\[ f_z = y \ln(xy) \]

Section 11.3, # 58: Determine the value of \( \partial x / \partial z \) at the point \((1, -1, -3)\) if the equation \(xz + y \ln x - x^2 + 4 = 0\) defines \(x\) as a function of the two independent variables \(y\) and \(z\).

The strategy is to use implicit differentiation: if the operator \(\partial / \partial z\) is applied, then \(x\) is a function of \(z\) (and \(y\)), and so \((\partial / \partial z)(x) = \partial x / \partial z\), but \((\partial / \partial z)(y) = 0\) since in a partial derivative, the other independent variables are held constant.

\[
\frac{\partial}{\partial z} \left( xz + y \ln x - x^2 + 4 = 0 \right)
\]

\[
\frac{\partial x}{\partial z} z + x \cdot 1 + y \cdot \frac{1}{x} \frac{\partial x}{\partial z} - 2x \frac{\partial x}{\partial z} + 0 = 0
\]

Solve the above for \(\partial x / \partial z\):

\[
\frac{\partial x}{\partial z} = \frac{-x}{z + (y/x) - 2x} = \frac{-x^2}{zx + y - 2x^2}
\]

Section 11.3, # 66: Show that \(f(x, y) = \ln \sqrt{x^2 + y^2}\) satisfies Laplace’s equation: \(f_{xx} + f_{yy} = 0\).

First, simplify the expression for \(f\) by using the fact that \(\ln a^b = b \ln a\):

\[
f(x, y) = \frac{1}{2} \ln(x^2 + y^2)
\]

\[
f_x = \frac{1}{2} \cdot \frac{2x}{x^2 + y^2} = \frac{x}{x^2 + y^2}
\]

\[
f_{xx} = \frac{(x^2 + y^2) \cdot 1 - x \cdot (2x + 0)}{(x^2 + y^2)^2} = \frac{y^2 - x^2}{(x^2 + y^2)^2}
\]
Since \( f(x, y) = f(y, x) \), it will be the case that \( f_{yy} \) has the same expression as \( f_{xx} \) except that the roles of \( x \) and \( y \) are switched. Thus,

\[
f_{yy} = \frac{x^2 - y^2}{(y^2 + x^2)}.
\]

It is now clear that \( f_{xx} + f_{yy} = 0 \).

Section 11.3, # 70: Show that \( w = \cos (2x + 2ct) \) is a solution to the wave equation: \( w_{tt} = c^2 w_{xx} \).

\[
\begin{align*}
w_x &= -2 \sin (2x + 2ct), & w_{xx} &= -4 \cos (2x + 2ct) \\
w_t &= -2c \sin (2x + 2ct), & w_{tt} &= -4c^2 \cos (2x + 2ct)
\end{align*}
\]

It is now clear that \( w_{tt} = c^2 w_{xx} \).