Selected Solutions to Homework # 4

1. Solve exercise # 16 in section 10.1.
   Compute the angle between the velocity and the acceleration vectors at time \( t = 0 \) where the position is given by
   \[ r(t) = \left( \frac{\sqrt{2}}{2} t \right) \mathbf{i} + \left( \frac{\sqrt{2}}{2} t - 16t^2 \right) \mathbf{j}. \]
   Compute the velocity and acceleration:
   \[ v(t) = \frac{\sqrt{2}}{2} \mathbf{i} + \left( \frac{\sqrt{2}}{2} - 32t \right) \mathbf{j}, \quad v(0) = \frac{\sqrt{2}}{2} \mathbf{i} + \frac{\sqrt{2}}{2} \mathbf{j}; \]
   \[ a(t) = -32 \mathbf{j}, \quad a(0) = -32 \mathbf{j}. \]
   To compute the angle \( \theta \) between these vectors, use
   \[ \cos \theta = \frac{v(0) \cdot a(0)}{|v(0)||a(0)|} = \frac{\frac{\sqrt{2}}{2}(-32)}{(1)(32)} = -\frac{\sqrt{2}}{2}. \]
   Therefore, \( \theta = \frac{3\pi}{4} \).

2. Solve exercise # 20 in section 10.1.
   Find parametric equations for the line tangent to the curve
   \[ r(t) = (2 \sin t) \mathbf{i} + (2 \cos t) \mathbf{j} + 5t \mathbf{k} \]
   at \( t_0 = 4\pi \).
   First compute the velocity vector:
   \[ v(t) = (2 \cos t) \mathbf{i} + (-2 \sin t) \mathbf{j} + 5 \mathbf{k} \]
   Then compute \( v(t_0) = 2 \mathbf{i} + 5 \mathbf{k} \) and \( r(t_0) = 2 \mathbf{j} + 20\pi \mathbf{k} \).
   The equation of the tangent line will be
   \[ L(t) = r(t_0) + t v(t_0) = 2t \mathbf{i} + 2 \mathbf{j} + (5t + 20\pi) \mathbf{k}. \]
   In parametric form, the line has equations
   \[ x(t) = 2t, \quad y(t) = 2, \quad z(t) = 5t + 20\pi. \]

   This problem was solved in class on 01/21/2011.