Solutions

LB 220 Calculus III, Quiz 4

Please answer in the space provided. Answers without supporting work may not receive full credit. You have 10 minutes to complete this quiz.

1. (3 points) Suppose that velocity of particle is given by \( \mathbf{v} = 2t \mathbf{i} + 3t^2 \mathbf{j} \). If the particle is at the point \( (1, 2) \) when \( t = 1 \), what is the position vector of the particle when \( t = 2 \)?

\[
\mathbf{r} = \int \mathbf{v} \, dt = t^2 \mathbf{i} + t^3 \mathbf{j} + \mathbf{C}
\]

\[
\mathbf{r}(1) = <1, 2> = <1, 1> + \mathbf{C}
\]

So, \( \mathbf{C} = <0, 1> \) and \( \mathbf{r} = <t^2, t^3> + <0, 1> \)

\[
\mathbf{r}(2) = <4, 8> + <0, 1> = <4, 9>
\]

2. (3 points) Determine the unit tangent vector \( \mathbf{T}(t) \) to the curve given by \( \mathbf{r}(t) = 2 \sin t \mathbf{i} + 3 \cos t \mathbf{j} \).

\[
\frac{d\mathbf{r}}{dt} = 2 \cos t \mathbf{i} - 3 \sin t \mathbf{j}
\]

\[
\left\| \frac{d\mathbf{r}}{dt} \right\| = \sqrt{4 \cos^2 t + 9 \sin^2 t}
\]

\[
\mathbf{T} = \frac{1}{\sqrt{4 \cos^2 t + 9 \sin^2 t}} \left< 2 \cos t, -3 \sin t \right>
\]
3. (3 points) What is the cosine of the angle between the velocity an acceleration vectors at $t = 0$ of the particle whose position is given by the vector-valued function shown below?

\[ r(t) = \frac{1}{1 + t^2} \mathbf{i} + \ln(1 + t) \mathbf{j} = \left\langle \left(1 + t^2\right)^{-1}, \ln(1 + t) \right\rangle \]

\[ \vec{v} = \frac{-2t}{(1 + t^2)^2} \mathbf{i} + \frac{1}{1 + t} \mathbf{j} \]

\[ \vec{v}(0) = \mathbf{j} \]

\[ \vec{a} = \frac{(1 + t^2)^2(-2) - (-2t)(2(1 + t^2)(2t))}{(1 + t^2)^4} \mathbf{i} + \frac{-1}{(1 + t)^2} \mathbf{j} \]

\[ \vec{a}(0) = \frac{-2 - 0}{(1)^4} \mathbf{i} + (-1) \mathbf{j} = -2 \mathbf{i} - \mathbf{j} \]

\[ \cos \theta = \frac{\vec{v}(0) \cdot \vec{a}(0)}{\|\vec{v}(0)\| \|\vec{a}(0)\|} = \frac{-1}{\frac{1}{\sqrt{5}}} = \frac{-1}{\sqrt{5}} \]