LB 220 Calculus III, Quiz 1

Please answer in the space provided. Answers without supporting work may not receive full credit. You have 10 minutes to complete this quiz.

1. (6 points) Consider the following three points in the plane: \(A(1, 1), B(4, 1),\) and \(C(6, 3).\) Let \(\vec{v}\) be the vector represented by the directed line segment \(\overrightarrow{AB}\) and \(\vec{w}\) by \(\overrightarrow{AC}.$

(a) Express \(\vec{v}\) and \(\vec{w}\) in component form.
\[
\vec{v} = \langle 4-1, 1-1 \rangle = \langle 3, 0 \rangle
\]
\[
\vec{w} = \langle 6-1, 3-1 \rangle = \langle 5, 2 \rangle
\]

(b) Express \(\vec{w} - 2\vec{v}\) in component form.
\[
\langle 5, 2 \rangle - 2 \langle 3, 0 \rangle
\]
\[
= \langle 5 - 6, 2 - 0 \rangle = \langle -1, 2 \rangle
\]

(c) Sketch \(\vec{v}, \vec{w},\) and \(\vec{w} - 2\vec{v}\) in standard position, meaning that each vector is represented by a directed line segment with its tail at the origin. Carefully label the axes so that it is clear that you sketch is correct.
2. (4 points) Consider the following points in space: \( A(1, 2, 3) \), \( B(2, 3, 1) \) and \( C(3, 1, 2) \).

(a) Sketch \( A \), \( B \), and \( C \), and label each point and each axis. Hint: Make your sketch rather large so that it does not look crowded.

(b) Let \( \mathbf{u} \) be the unit vector in the same direction as \( \overrightarrow{AB} \). Write \( \mathbf{u} \) in component form. You do not need to rationalize the denominator.

\[
\overrightarrow{AB} = \left< 2 - 1, 3 - 2, 1 - 3 \right> = \left< 1, 1, -2 \right>
\]

\[
\|\overrightarrow{AB}\| = \sqrt{1^2 + 1^2 + (-2)^2} = \sqrt{6}
\]

\[
\mathbf{u} = \left< \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}} \right>
\]