LB 119 Homework 3 (due Wednesday, 09/19/12)

Directions. Please work on the problems below. Your solutions must begin with a clear statement (or re-statement in your own words) of the problem. Your solutions should be clear, legible, and demonstrate at minimum partial progress towards a complete solution to the problem. Please refer to the syllabus for the policy on grading (communication, completeness, and correctness) and late homework (late homework is not accepted; homework is collected during class).

Collaboration. I encourage you to discuss the homework problems with your classmates. However, each student must write and submit his or her own homework solutions.

1. Determine the centroid the region in the plane which lies above the $x$-axis and below the semi-circle $y = \sqrt{r^2 - x^2}$.

2. The following principle is used frequently in physics: if system $S$ of point masses is given, then we can regard the system as a single point mass located at the center of mass of $S$ and having mass equal to the total mass of $S$. For example, a large asymmetric object might, for simplicity, be regarded as only occupying a single point in space and that the mass of this large object might be regarded as concentrated at this single point. The natural candidate for such a point is the center of mass of the object. In this exercise, you will provide a mathematical justification for why this is “natural”.

Let $S$ be the system of point masses $p_1, \ldots, p_k$, and let $T$ be the system of point masses $p_{k+1}, \ldots, p_n$. Suppose that each point mass $p_i$ has mass $m_i$ at that each that each point $p_i$ lies on the real line and has $x$-coordinate equal to $x_i$. Prove that the center of mass of the system $S \cup T$ of consisting of all of the point masses, $p_1, \ldots, p_n$, is equal to the center of mass of the system consisting of two special point masses: one point mass $p_S$ which has a mass equal to the total mass of the system $S$ and which has $x$-coordinate equal to the center of mass of the system $S$ and a second point mass $p_T$ defined analogously with respect to the system $T$.

3. In this exercise you will use the principle above to determine the centroid of pentagon which is shaped like a child’s drawing of a
house. Let $A, B, C, D$ and $E$ be points in the plane such that the quadrilateral $ABCD$ is a square with side length $s$ and such that $E$ lies outside this square and such that $AEB$ is a triangle such that $\angle AEB$ is a right triangle; let $P$ be the pentagon having the above vertices. First determine the centroid of the square and the centroid of the triangle; then use the principle in the previous exercise to determine the centroid of $P$. (You will need to choose coordinates for the vertices; try to make choices which simplify your calculations. You should prefer arguments which take advantage of the symmetry to arguments which use brute force calculations.)

**Reminder:** You should be working through the recommended textbook exercises. A list of these problems can be found on our course web page: