LB 119 Homework 2 (due Wednesday, 09/12/12)

**Directions.** Please work on the problems below. Your solutions must begin with a clear statement (or re-statement in your own words) of the problem. Your solutions should be clear, legible, and demonstrate at minimum partial progress towards a complete solution to the problem. Please refer to the syllabus for the policy on grading (communication, completeness, and correctness) and late homework (late homework is not accepted; homework is collected during class).

**Collaboration.** I encourage you to discuss the homework problems with your classmates. However, each student must write and submit his or her own homework solutions.

1. Use a cross section method (e.g. disk, washer, or shell method – your choice) to demonstrate that the volume of a right circular cone of radius $r$ and height $h$ is equal to $\frac{\pi}{3} r^2 h$.

2. Suppose that $S$ is a planar region enclosed by a square having an edge of length $l$, that $C$ is a planar region enclosed by a circle having radius $r$, and that $H$ is a planar region enclosed by a regular hexagon having an edge of length $l$. In each case, compute the ratio of the area of the region to the area of the region obtained by scaling it by a factor of $c$, where $c > 0$. For example, if $c = 1/2$, then to scale $S$ by a factor of $c$ means that its edges now have length $l/2$. Sketch an example in each case (square, circle, regular hexagon) of such a region and scaled copy of each region. Your sketch should be accurate (measure the distances) and you should state which value of $c$ you chose for your sketches.

Suppose that $R$ is a planar figure enclosing a finite area. What do you conjecture is the ratio of the area of $R$ to the area of the figure obtained by scaling $R$ by a factor of $c$, where $c > 0$?

3. Let $R$ be the triangular region in the plane enclosed by the lines $y = 0$, $y = x - 1$ and $y = 3 - x$. Let $S$ be the solid of revolution generated by rotating the region $R$ about the $y$-axis. Compute the volume of $S$ in two different ways: once using the washer method and once using the shell method. Which of these two methods seems more efficient? Why? Finally, compute the volume in a third way by
applying The Theorem of Pappus (Theorem 7.1 in Section 7.6); you do not need to show any work for the computation of the centroid—just state what it is—the answer is geometrically obvious.

4. Let $a$ and $c$ be positive constants. Let $L$ be the locus of points in the plane whose coordinates $(x, y)$ satisfy $y = c\sqrt{x}$ and $0 \leq x \leq a$.
Determine the surface area of the surface of revolution generated by revolving the curve $L$ about the $x$-axis.

5. Suppose that a lamina, i.e. a thin plate, is described as the region $R$ in the plane enclosed by a circle of radius $r$ centered at the origin. Suppose further that the density of the lamina is given as a function of the $x$-coordinate of the points in $R$ by the function $\delta(x) = 1 + |x|$. Compute the mass of the lamina. Is it necessary in this case to first compute the mass if your objective was to determine the center of mass of the lamina? Explain.