Math 950  Homework 2  Due 02/19/09

Prob 1 (5pt). (nonlinear pendulum)

(a) Write a program to solve the boundary value problem for the nonlinear pendulum as discussed in the text. See if you can find yet another solution for the boundary conditions illustrated in Figures 2.4 and 2.5.

(b) Find a numerical solution to this BVP with the same general behavior as seen in Figure 2.5 for the case of a longer time interval, say $T = 20$, again with $\alpha = \beta = 0.7$. Try larger values of $T$. What does $\max_i \theta_i$ approach as $T$ is increased? Note that for large $T$ this solution exhibits “boundary layers”.

Prob 2 (5pt). (9-point Laplacian)

(a) Show that the 9-point Laplacian (3.17) has the truncation error derived in Section 3.5. Hint: To simplify the computation, note that the 9-point Laplacian can be written as the 5-point Laplacian (with known truncation error) plus a finite difference approximation that models $\frac{1}{6} h^2 u_{xxyy} + O(h^4)$.

(b) Modify the matlab script poisson.m to use the 9-point Laplacian (3.17) instead of the 5-point Laplacian, and to solve the linear system (3.18) where $f_{ij}$ is given by (3.19). Perform a grid refinement study to verify that fourth order accuracy is achieved.