Show intermediate steps!

1. (10pt) Use the definition of the derivative as a limit to calculate $f'(x)$ for $f(x) = \frac{2x}{x-1}$.
(No credit for other methods)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2(x+h)}{(x+h)-1} - \frac{2x}{x-1}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2(x+h)(x-1) - 2x(x+h-1)}{(x+h-1)(x-1)}}{h}$$

$$= \lim_{h \to 0} \frac{2x(x+h) - 2x - 2x(x+h) + 2x}{h(x+h-1)(x-1)}$$

$$= \lim_{h \to 0} \frac{-2h}{h(x+h-1)(x-1)}$$

$$= \lim_{h \to 0} \frac{-2}{(x+h-1)(x-1)}$$

$$= \frac{-2}{(x-1)^2}$$
2. (10pt) A function \( y = y(x) \) is defined implicitly by the equation
\[
x^2 + xy^2 + y + 1 = 0
\]
Find derivative \( \frac{dy}{dx} \) as a function of \( x \) and \( y \). Write the tangent line at point \((-1, -1)\).

Differentiate on both sides:
\[
2x + 2x\frac{dy}{dx} + 2y\frac{dy}{dx} + y' = 0
\]
\[
\Rightarrow (2x + 2y) \frac{dy}{dx} = -2x - y^2
\]
\[
\Rightarrow \frac{dy}{dx} = \frac{-2x - y^2}{2x + 2y + 1}
\]

At \((-1, -1)\),
\[
\frac{dy}{dx} = \frac{-2(-1) - (-1)^2}{2(-1) + 2(-1) + 1} = \frac{1}{3}
\]

Tangent line
\[
y - (-1) = \frac{1}{3}(x - (-1))
\]
or
\[
y = \frac{1}{3}x - \frac{2}{3}.
\]
3. (10pt) A spherical balloon is inflated at a rate of 10 cubic inches per second. How fast is the surface area of the balloon increasing when the radius is 2 inches long? Hint: The volume of a sphere of radius \( r \) is given by \( V = \frac{4}{3}\pi r^3 \) and the surface area is given by \( S = 4\pi r^2 \).

\[
\frac{dV}{dt} = 10 \text{ in}^3/\text{sec}.
\]

\[
= 4\pi \cdot r^2 \cdot \frac{dr}{dt}.
\]

\[
\text{At } r = 2, \quad \frac{dr}{dt} = \frac{5}{8\pi} \text{ in}/\text{sec}.
\]

\[
\frac{dS}{dt} = 8\pi r \cdot \frac{dr}{dt}
\]

\[
= 10\pi \text{ in}^2/\text{sec}.
\]
4. (10pt) Find the linearization of the function \( f(x) = \sqrt{x+2} \) at the point \( a = 2 \). Then using the linearization you have found, estimate \( \sqrt{4.1} \).

\[
\frac{df}{dx} = \frac{1}{2} (x+2)^{-1/2}
\]

At \( a = 2 \),

\[
f'(a) = \frac{1}{2} \cdot 2 = 1
\]

\[
f'(a) = \frac{1}{2} \cdot (2+2)^{-1/2} = \frac{1}{4}.
\]

\[
L(x) = f(a) + f'(a)(x-a)
= 2 + \frac{1}{4} (x-2)
\]

\[
\sqrt{4.1} = f(2.1) \approx L(2.1) = 2 + \frac{1}{4} (2.1-2)
= 2 + \frac{0.1}{4}
= 2.025.
\]