Math 421 / Homework 9.3

 # 2 Compute the iterated limits at $(0,0)$ of each of the following functions. Determine which of these functions has a limit as $(x,y) \to (0,0)$ in $\mathbf{R}^2$, and prove that the limit exists.

(a) $f(x,y) = \dfrac{\sin x \sin y}{x^2+y^2}$

(b) $f(x,y) = \dfrac{x^2+y^4}{x^2+2y^4}$

(c) $f(x,y) = \dfrac{x-y}{(x^2+y^2)^\alpha}$, where $\alpha < 1/2$.

 # 3 Prove that each of the following functions has a limit as $(x,y) \to (0,0)$ in $\mathbf{R}^2$.

(a) $f(x,y) = \dfrac{x^3-y^3}{x^2+y^2}$, $(x,y) \neq (0,0)$

(b) $f(x,y) = \dfrac{|x|^\alpha y^4}{x^2+y^4}$, $(x,y) \neq (0,0)$, where α is any positive number.

 # 5 Suppose that $\mathbf{a} \in \mathbf{R}^n$, that $\mathbf{L} \in \mathbf{R}^m$, and that $\mathbf{f}\colon \mathbf{R}^n \to \mathbf{R}^m$. Prove that if $\mathbf{f}(\mathbf{x}) \to \mathbf{L}$ as $\mathbf{x} \to \mathbf{a}$, then there is an open set V containing $\mathbf{a}$ and a constant $M > 0$ such that $\|\mathbf{f}(\mathbf{x})\| \leq M$ for all $\mathbf{x} \in V$.