Math 421 / Homework 9.1

 # 2 Using limit theorems, find the limit of each of the following vector sequences.

(a) $\mathbf{x}_k = \left(\frac{1}{k}, \frac{2k^2 - k + 1}{k^2 + 2k - 1}\right)$

(b) $\mathbf{x}_k = \left(1, \sin(\pi k), \cos\frac{1}{k}\right)$

(c) $\mathbf{x}_k = \left(k - \sqrt{k^2 + k}, k^{1/k}, \frac{1}{k}\right)$

 # 3 Suppose that $\mathbf{x}_k \to 0$ in $\mathbf{R}^n$ as $k \to \infty$ and that $\mathbf{y}_k$ is bounded in $\mathbf{R}^n$.

(a) Prove that $\mathbf{x}_k \cdot \mathbf{y}_k \to 0$ as $k \to \infty$.

(b) If $n = 3$, prove that $\mathbf{x}_k \times \mathbf{y}_k \to 0$ as $k \to \infty$.

 # 6 Let E be a nonempty subset of $\mathbf{R}^n$.

(a) Show that a sequence $(\mathbf{x}_k)$ in E converges to some point $\mathbf{a} \in E$ if and only if for every set U which is relatively open in E and contains $\mathbf{a}$, there exists an $N \in \mathbf{N}$ such that $\mathbf{x}_k \in U$ for $k \geq N$.

(b) Prove that a set $C \subseteq E$ is relatively closed in E if and only if whenever $\mathbf{x}_k \in C$ with $(\mathbf{x}_k) \to \mathbf{a} \in E$ it follows that $\mathbf{a} \in C$.